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THESIS

MEASUREMENT OF TURBOFAN/TURBOJET THRUST
FROM TAILPIPE STATIC PRESSURE

by

Todd Williams Givens

and

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December 1984

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It was decided to approach the data evaluation empirically. The engine data were combined and an ensemble plot of tailpipe static pressure versus thrust was produced for analysis. A curve fitting technique was then employed to determine how well the parameter correlated with thrust.

The results were tested statistically and found to be reasonable. Correlation between thrust and tailpipe static pressure was excellent.

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Measurement Of Turbofan/Turbojet Thrust
From Tailpipe Static Pressure

by

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December 1984

ABSTRACT

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TF41 engine data were provided by the Naval Air Rework Facility at Jacksonville, Florida. The data consisted of a variety of engine parameters which had been recorded during routine post-maintenance performance tests plus an additional set of tailpipe static pressure readings that had been obtained from a "slave" tailpipe used for this project.

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This task was initiated by a contract with the Naval Air Engineering Center (NAEC) under the direction of Dr. A. Elshinnawy.

LIST OF SYMBOLS/ABBREVIATIONS

S/N	Serial Number
ps6	Tailpipe Static Pressure
ps6c	ps6 Corrected for Standard Conditions
m	Mass Flow Rate
ρ	Density
γ	Ratio of Specific Heats
p or p	Static Pressure
r	Sample Correlation Coefficient
rho	Population Correlation Coefficient
pt5.1	Stagnation Pressure @ Station 5:1 (See Figure D2)
WFC	Corrected Fuel Flow
WAIC	Corrected Airflow
NLC	Corrected Speed, Low Press. Spool
NHC	Corrected Speed, High Press. Spool

I. INTRODUCTION

The accurate measurement of uninstalled thrust for a turbojet or turbofan aircraft engine is best accomplished mechanically in a properly calibrated, well-instrumented, environmentally controlled test cell. This technique suffers, however, from a rather high degree of sophistication (which translates directly to high cost) and a lack of portability. Further, if a large number of engines are to be measured, the extensive time involved in transporting, hook-up, etc. can profoundly impact maintenance turn-around-time and, therefore, operational readiness.

Thrust can also be measured indirectly; that is, by measuring other parameters and inferring the thrust from those measurements. Intuitively, the accuracy of this technique will depend on the accuracy of the measurements and on the ability of a mathematical model to correctly represent the thrust. Tests have been conducted by the National Aeronautics and Space Administration (NASA) and the United States Air Force (USAF) to verify the accuracy of in-flight thrust measurement techniques, with promising results. Of particular interest are the results of tests of a "simplified gross thrust calculation technique" conducted by NASA using the F100 augmented turbofan engine [Refs. 1, 2]. These tests concluded that a model based on empirically

corrected, ideal, one-dimensional thermodynamic relationships could predict in-flight thrust with an uncertainty of less than three percent, using only four pressure measurements.

Our project sought to investigate the feasibility of employing a relatively unsophisticated, and therefore low cost, indirect method for measuring uninstalled thrust with acceptable accuracy. Since aircraft inlet flow distortion and most other flow interference effects are absent from the uninstalled thrust measurement problem, it seemed reasonable to assume that measuring uninstalled thrust indirectly could be accomplished more accurately than measuring in-flight thrust, and/or fewer measured parameters would be needed. To simplify the problem somewhat, a turbofan engine without augmentation was chosen as the test candidate (TF41).

Performance tests of a randomly selected group of TF41 engines were conducted at the Naval Air Rework Facility (NARF), Jacksonville, Florida in conjunction with routine post-maintenance evaluations. These engines were evaluated in accordance with the requirements of reference 3 with a modified tailpipe to provide tailpipe static pressure data in addition to the data routinely produced (Appendix D). All data were then forwarded to the Naval Postgraduate School for analysis.

II. NATURE OF THE PROBLEM

Several analytical techniques for measuring and/or calculating thrust that have been utilized in past efforts were evaluated. For the purposes of this project, however, they all suffer from the same deficiency. All require the measurement of more parameters than desired. Further, judging from past efforts, the models would require considerable empirical "correcting" before acceptable accuracy could be achieved. As an example, utilization of "a very complex gas generator method (GGM)" [Ref. 1] for evaluating the thrust of the F100 engine required measuring ten parameters (including fan speed, IGV position, area, pressures, temperatures and airflows); and "calibration" was still required.

It was decided, therefore, to approach the data evaluation empirically at the outset. Since it can be shown (by a first order analysis) that thrust varies directly with tailpipe static pressure (see Appendix A), the engine data were combined and an ensemble plot of tailpipe static pressure (ps6) versus thrust was produced for analysis. A curve fitting technique was then employed to determine how well the parameter correlated with thrust and an error analysis was conducted in order to evaluate the uncertainty associated with the regression (see Appendix B).

III. RESULTS

The model resulting from the initial regression analysis (see Appendix B) is shown in Figure 1 and the plot of residuals is shown in Figure 2. The correlation coefficient resulting from this analysis is excellent, but further investigation was indicated in order to evaluate two features of the results.

First, two outliers are readily apparent from examination of Figure 2 and an explanation was sought for such radical departures from an otherwise well-behaved distribution. Secondly, the data in Figure 1 are an ensemble of data points from nine different engines. The behavior of individual engines was considered to be of interest in order to gain at least a qualitative feel for the repeatability of the data.

To accomplish this further analysis, each engine was evaluated separately and the results compared. Although the number of data points was necessarily reduced (to an average of eight), the results, as presented in Table 1, show strong correlation for each engine and good agreement among engines, with two exceptions.

Engines S/N 142633 and 141257 differ slightly from the others either in the correlation coefficient (142633) or in the regression equation itself (141257).

TF41 ENGINE

LINEAR REGRESSION MODEL

CORRECTED THRUST VS. CORRECTED PS6

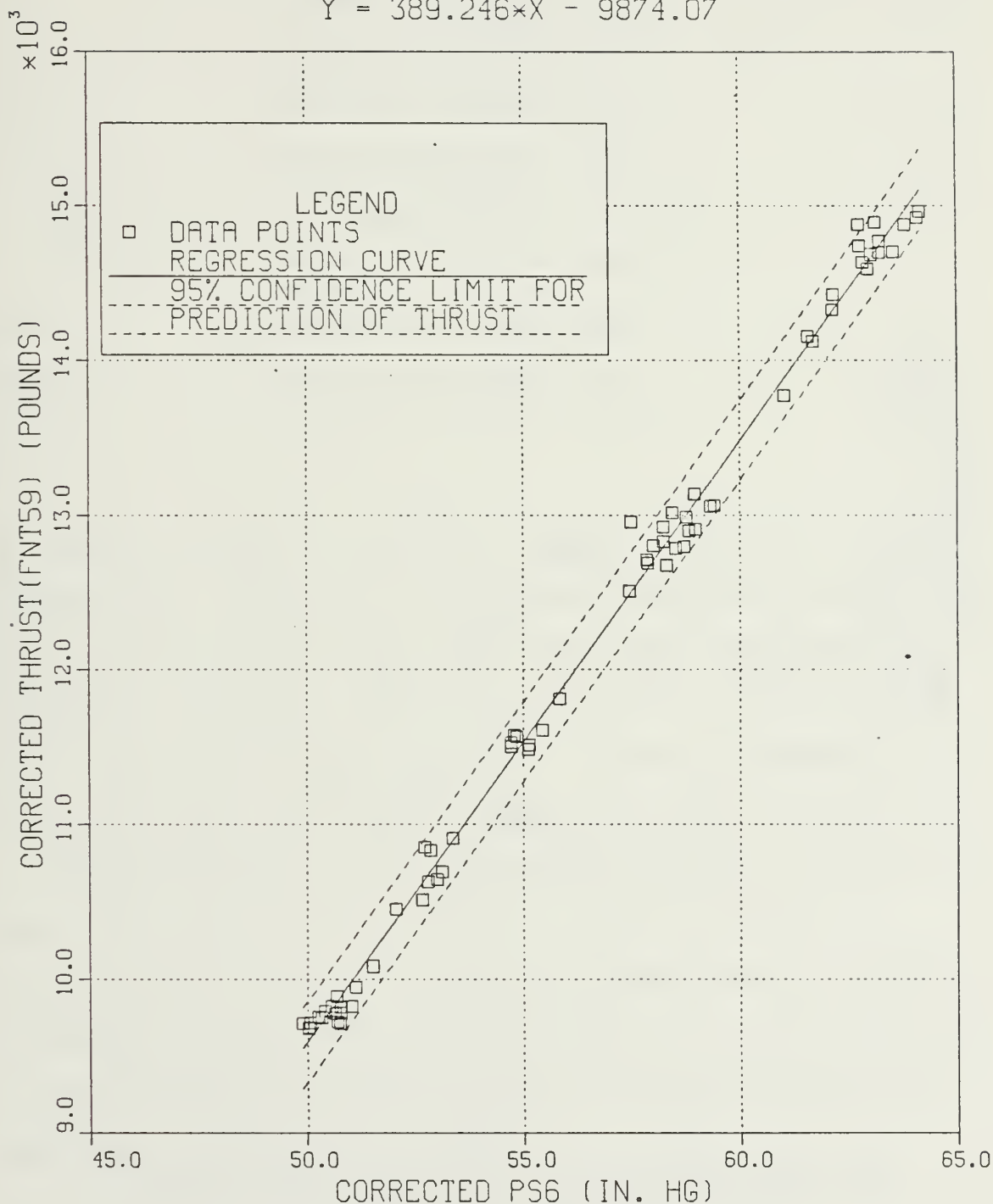
$$Y = 389.246 \times X - 9874.07$$


Figure 1 Thrust vs. ps6

TF41 ENGINE
 LINEAR REGRESSION MODEL
 RESIDUALS VS. PS6

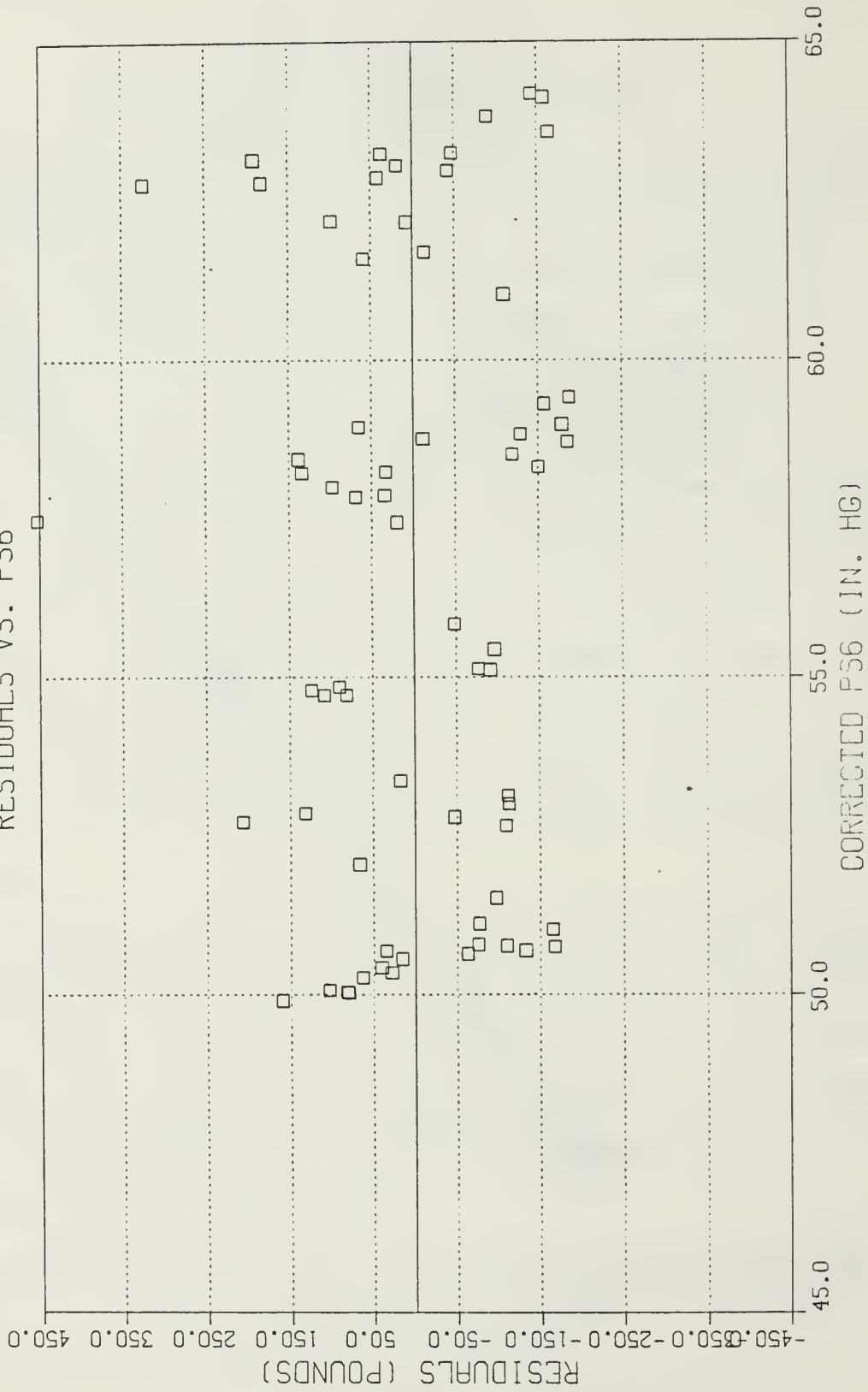


Figure 2 Residuals vs. ps6

TABLE 1

ENGINE S/N	REGRESSION EQUATION	r
141481	$FNT(59) = 386.4 * PS6C - 9623.9$	0.9994
141525	$FNT(59) = 386.7 * PS6C - 9680.9$	0.9999
142634	$FNT(59) = 391.3 * PS6C - 9843.3$	0.9999
141954	$FNT(59) = 390.0 * PS6C - 9823.2$	0.9996
141427	$FNT(59) = 387.5 * PS6C - 9912.8$	0.9998
141972	$FNT(59) = 385.9 * PS6C - 9814.8$	0.9999
141440	$FNT(59) = 394.9 * PS6C - 10245.6$	0.9998
142633	$FNT(59) = 386.8 * PS6C - 9661.8$	0.9967
141257	$FNT(59) = 404.1 * PS6C - 10766.5$	0.9977

Plots of these two engines are shown in Figures 3 and 4 respectively. Note that the regression equation plotted in each case excludes the two points that are highlighted. Upon their exclusion, these two engines are brought in line with the rest, as shown in Table 2. These two points are the same outliers as noted in the residuals plot, Figure 2.

Since the data had been recorded by hand and transcribed onto a hand written data sheet, it seemed plausible to assume that an error or two could have been made, and since exclusion of these two points resulted in "improved behavior" of their respective engines, these two outliers were also excluded from the overall analysis.

TF41 ENGINE S/N 142633
LINEAR REGRESSION MODEL
CORRECTED THRUST VS. CORRECTED PS6
 $Y = 385.2 \times X - 9635.2$

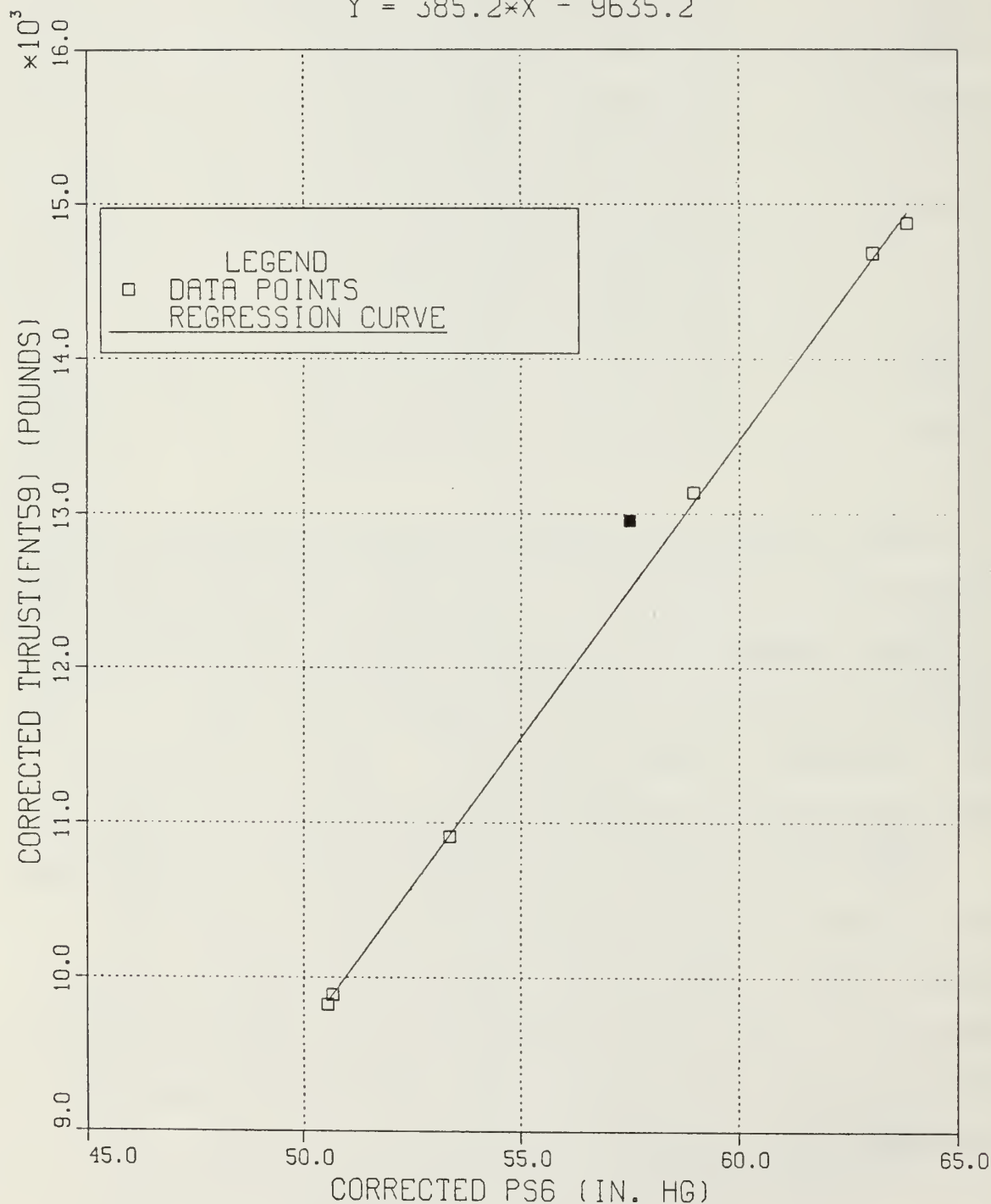


Figure 3 Engine S/N 142633 Line

TF41 ENGINE S/N 141257
 LINEAR REGRESSION MODEL
 CORRECTED THRUST VS. CORRECTED PS6
 $Y = 389.1 \times X - 9966.9$

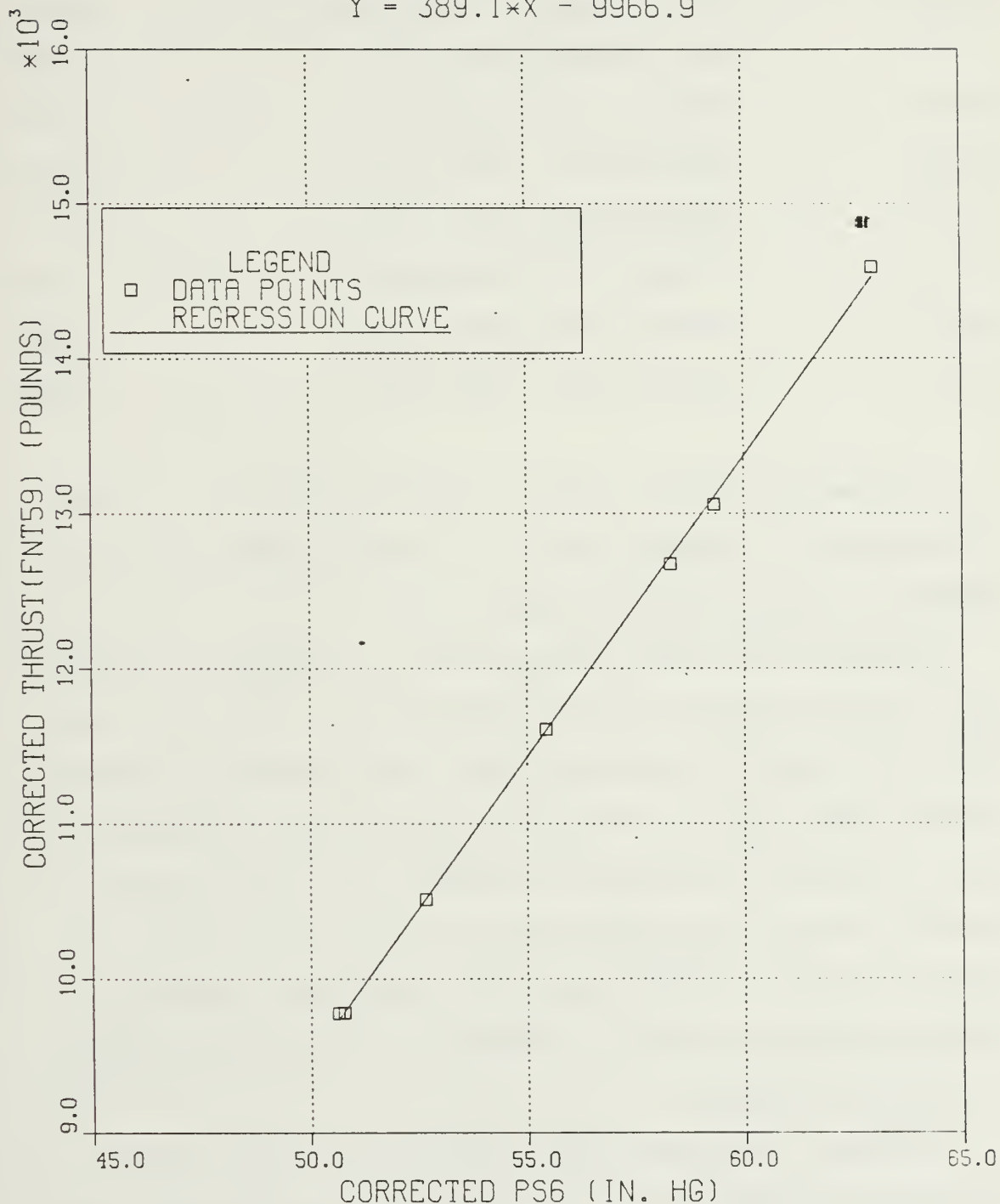


Figure 4 Engine S/N 141257 Line

TABLE 2

ENGINE S/N	REGRESSION EQUATION	r
141481	$FNT(59) = 386.4 * PS6C - 9623.9$	0.9994
141525	$FNT(59) = 386.7 * PS6C - 9680.9$	0.9999
142634	$FNT(59) = 391.3 * PS6C - 9845.3$	0.9999
141954	$FNT(59) = 390.0 * PS6C - 9823.2$	0.9996
141427	$FNT(59) = 387.5 * PS6C - 9912.8$	0.9998
141972	$FNT(59) = 385.9 * PS6C - 9814.8$	0.9999
141440	$FNT(59) = 394.9 * PS6C - 10245.6$	0.9998
142633	$FNT(59) = 385.2 * PS6C - 9635.2$	0.9998
141257	$FNT(59) = 389.1 * PS6C - 9966.9$	0.9997

A revised regression model, excluding the two outliers, is presented in Figures 5 and 6. A corresponding plot of residuals is presented in Figure 7.

Additional data were provided for a "correlation engine" which had been used to verify calibration of the test cell. These data were not included in the above because a pressure rake had been placed in the inlet area which, presumably, would influence the pressure readings within the engine. A separate analysis was conducted on those data and the results are presented in Figure 8. Note that, again, a strong correlation exists, although the regression equation is somewhat altered.

Reference 3 requires that the NARF plot other engine parameters against thrust to verify engine performance after

rework. As an indication that the model presented in Figure 5 or 6 could be effectively used, these parameters were also plotted against predicted thrust for comparison. These plots are included as Figures 9 through 18.

TF41 ENGINE

LINEAR REGRESSION MODEL

CORRECTED THRUST VS. CORRECTED PS6

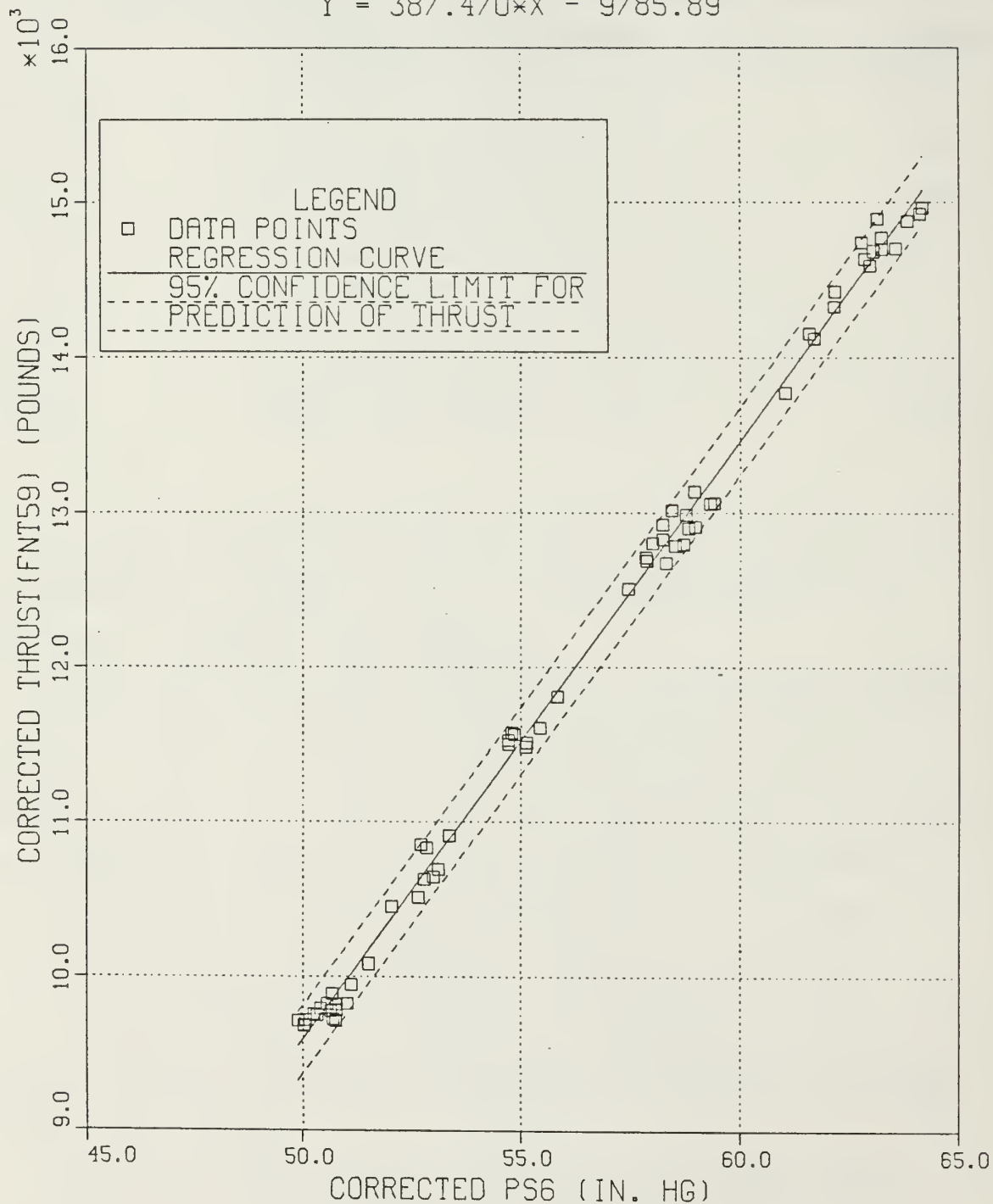
$$Y = 387.470 \times X - 9785.89$$


Figure 5 Thrust vs. ps6 (Revised)

TF41 ENGINE
 LINEAR REGRESSION MODEL
 CORRECTED THRUST VS. CORRECTED PS6
 $Y = 387.470 \times X - 9785.89$

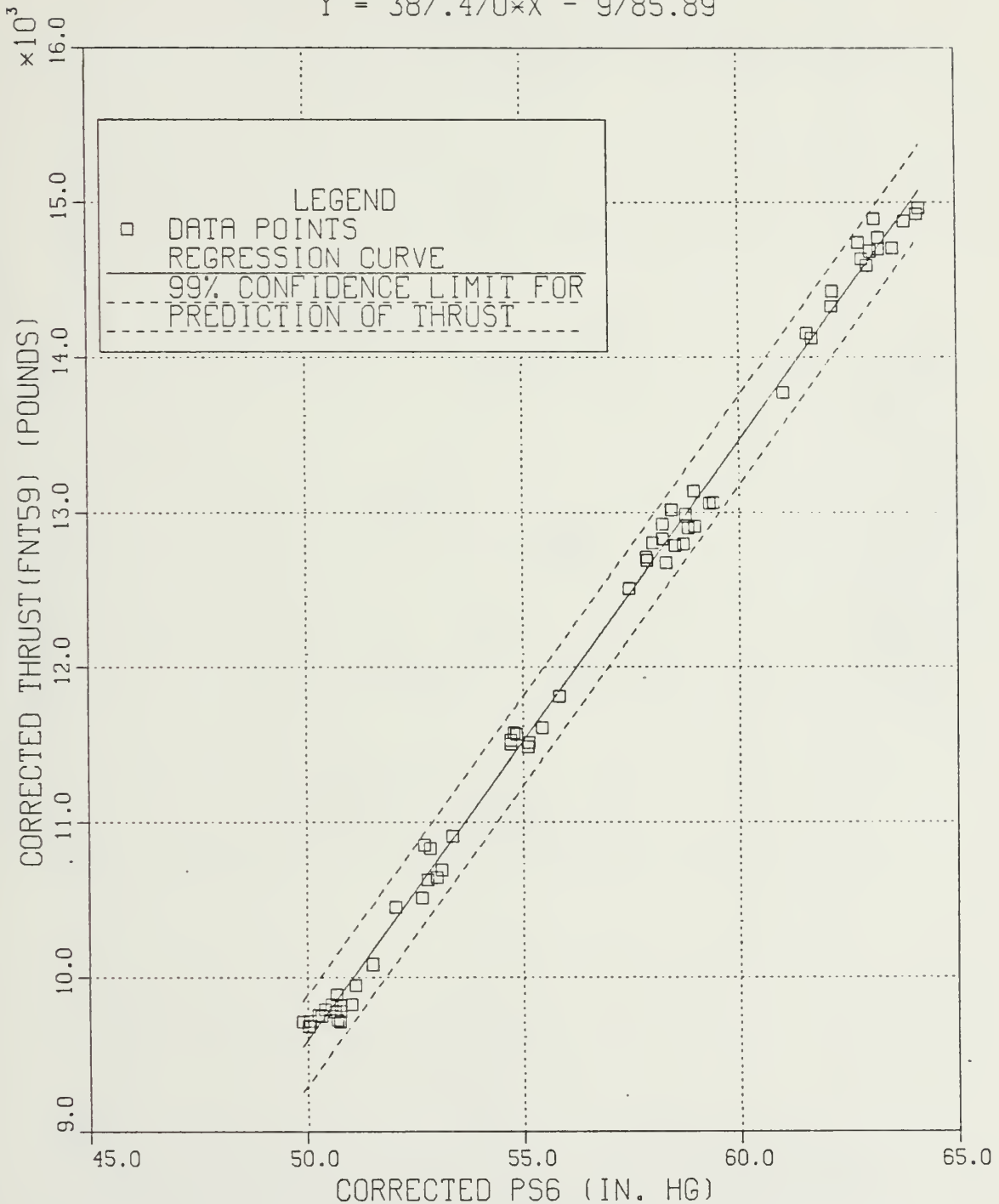


Figure 6 Thrust vs. ps6 (Revised, 99% Conf.)

TF41 ENGINE
 LINEAR REGRESSION MODEL
 RESIDUALS VS. PS6

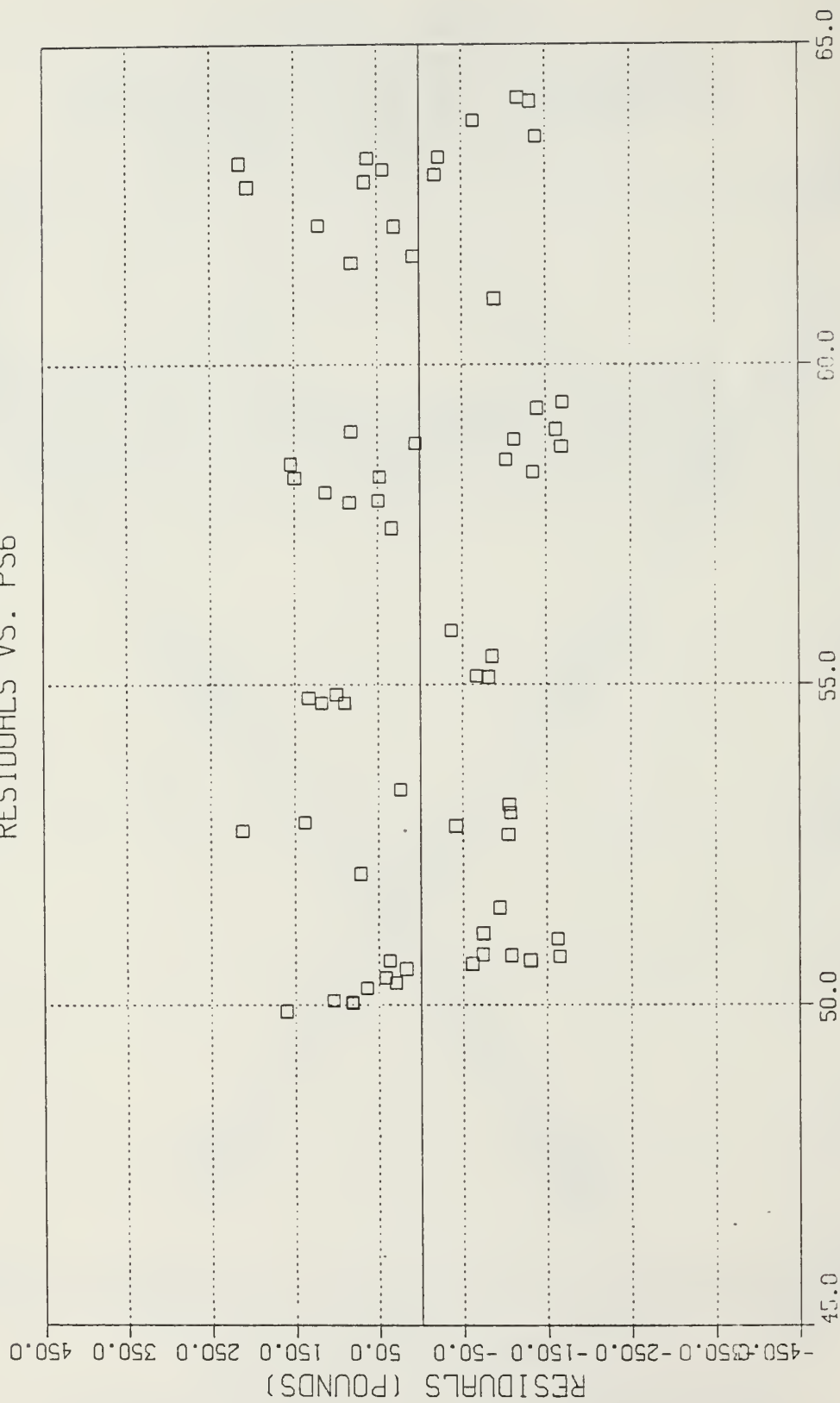


Figure 7 Residuals vs. ps6 (Revised)

TF41 ENGINE S/N 142618
 LINEAR REGRESSION MODEL
 CORRECTED THRUST VS. CORRECTED PS6
 $Y = 380.303 \times X - 9393.25$

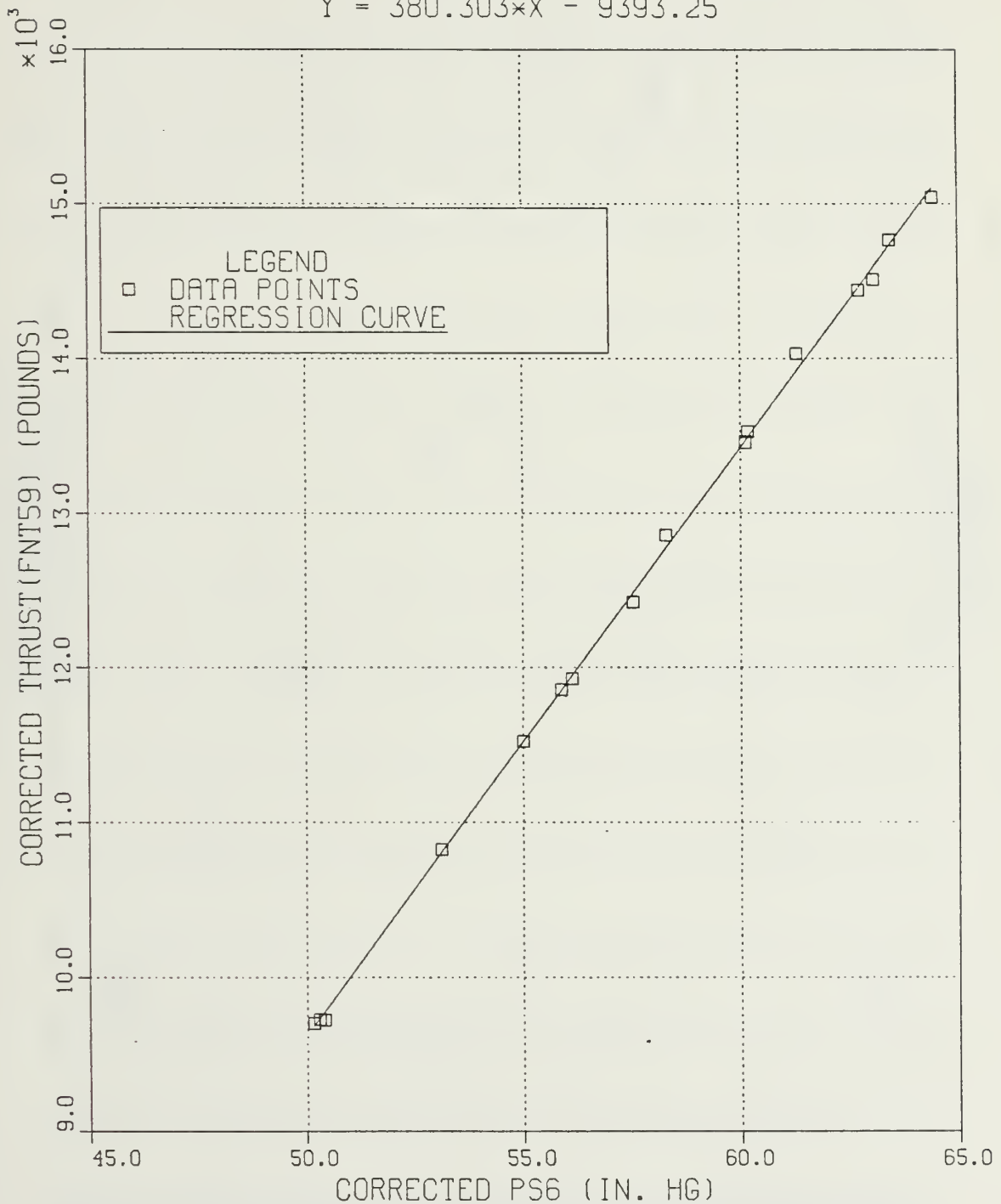


Figure 8 Engine S/N 142618 Line

TF41 ENGINE

PT5.1 VS. THRUST

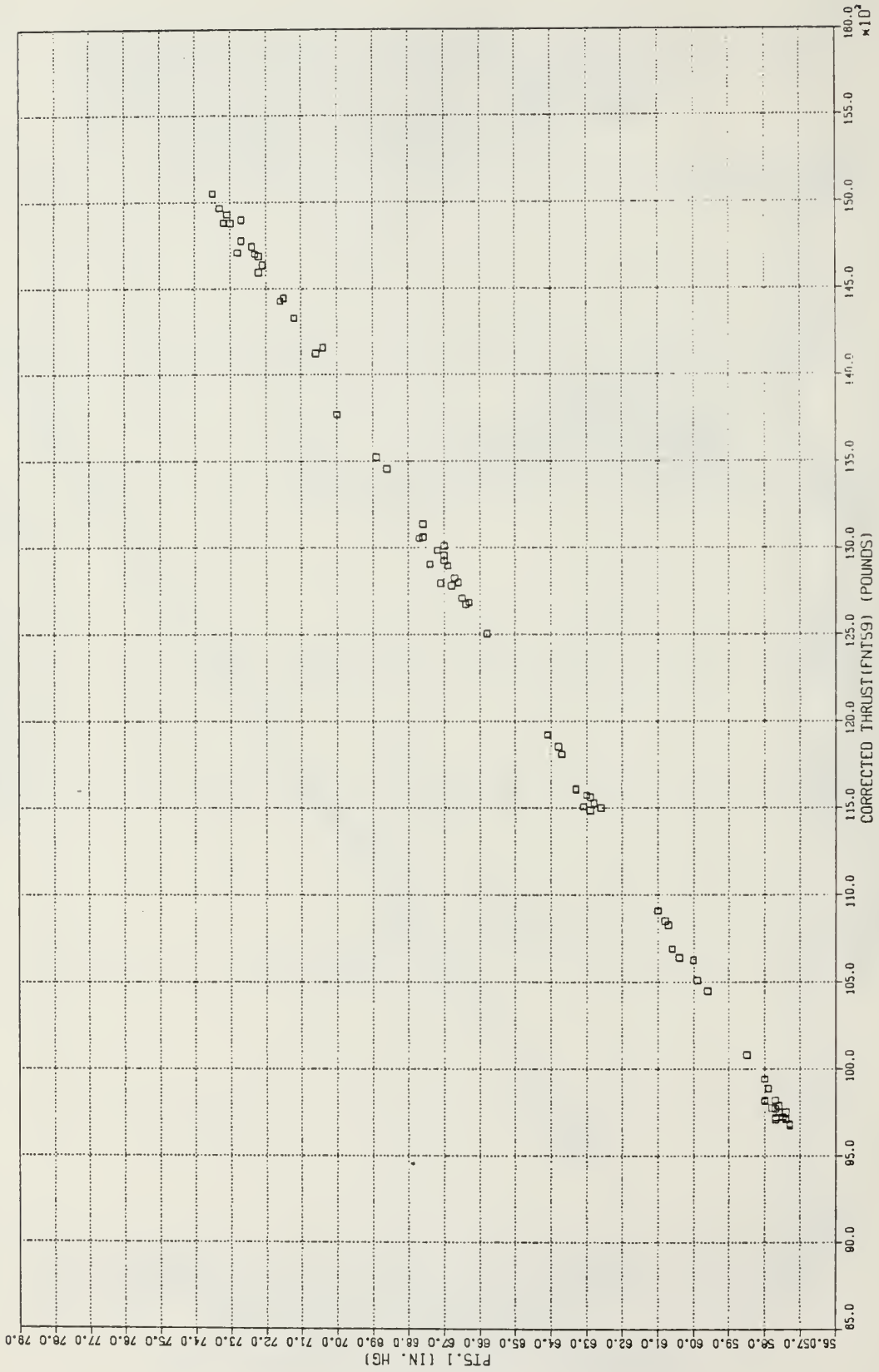


Figure 9 pt5.1 vs. Thrust

TF41 ENGINE
PT5.1 VS. THRUST
THRUST CALCULATED FROM REGRESSION MODEL
THRUST - 387.470*(P56) - 9785.89

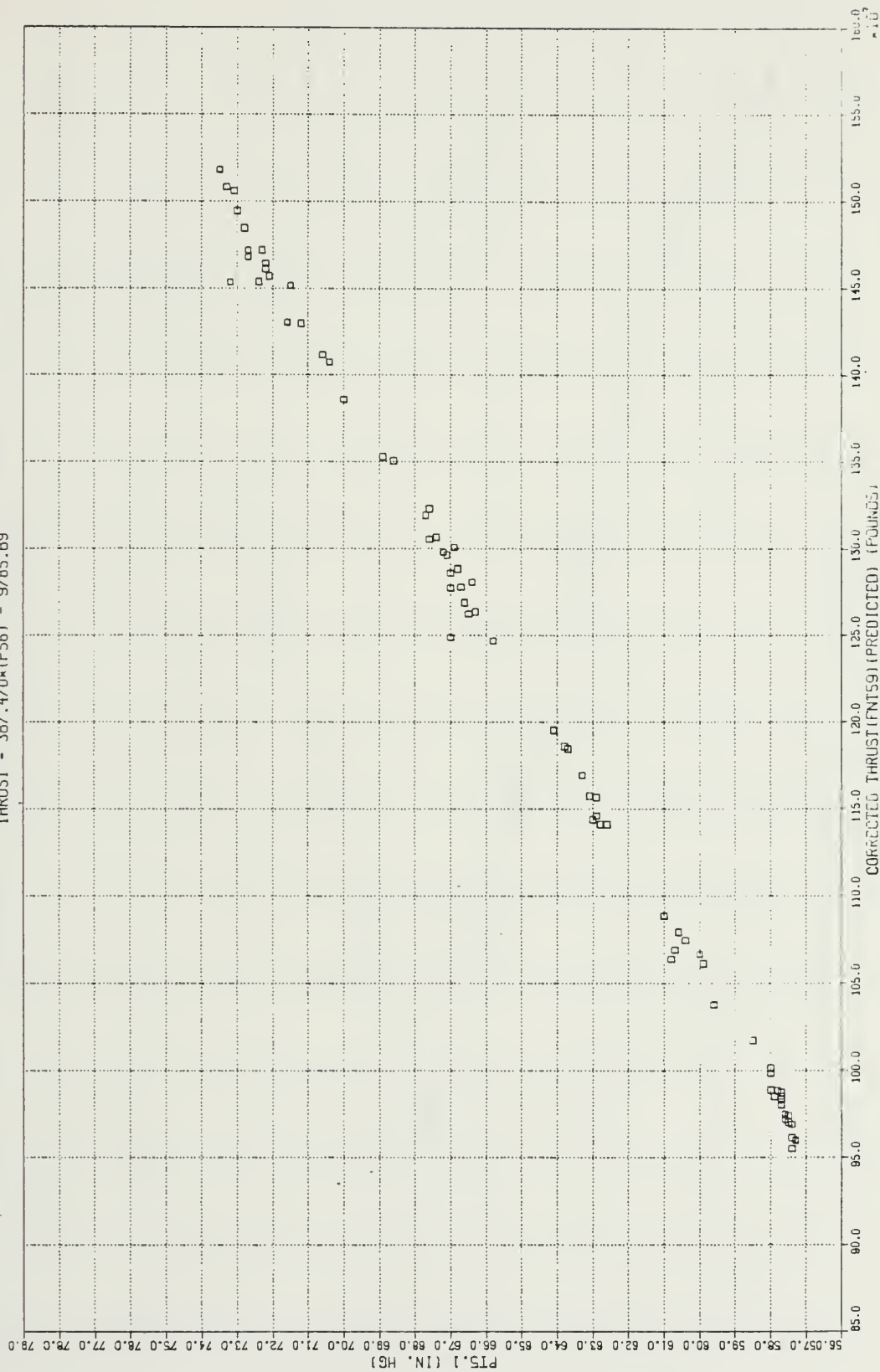
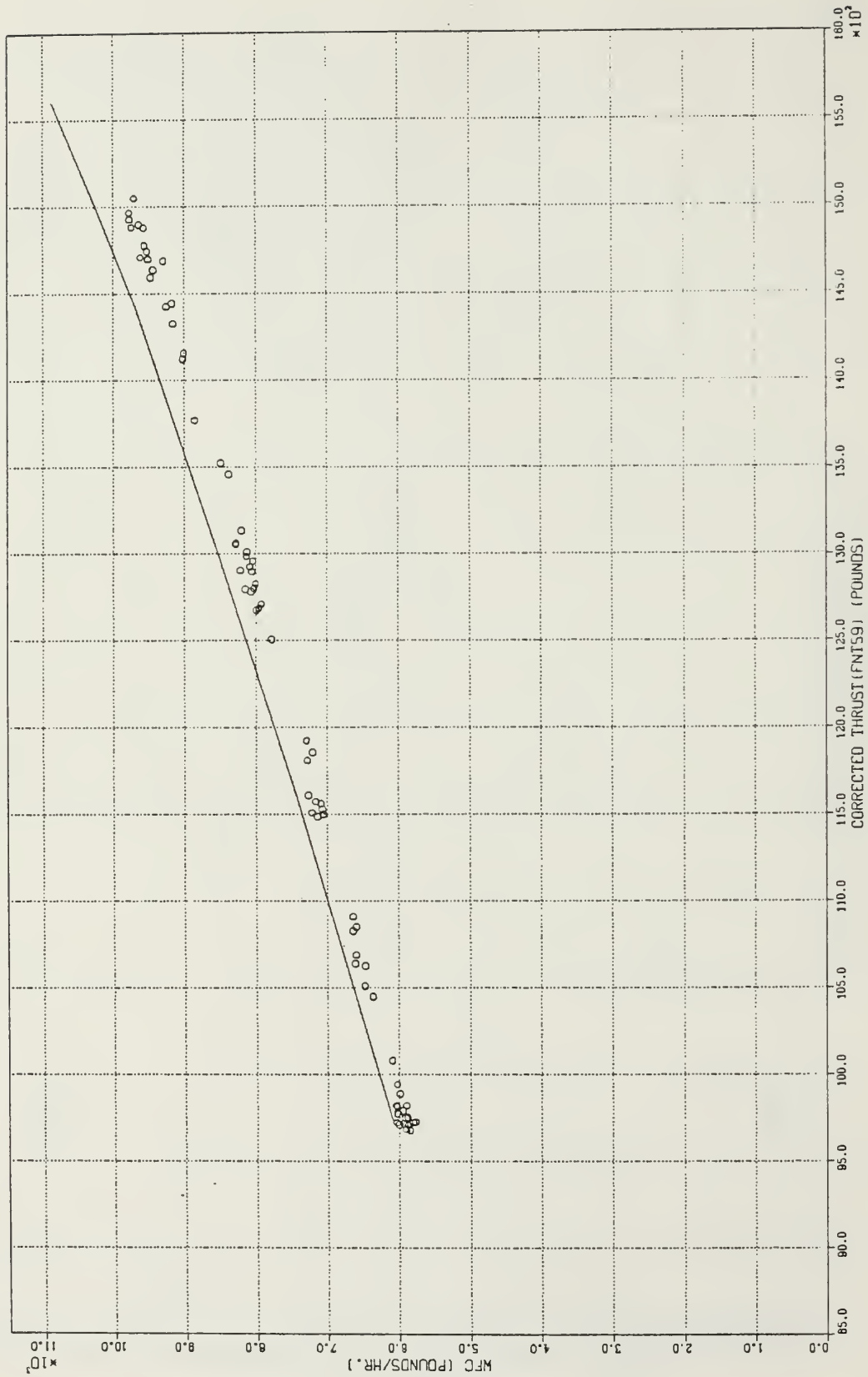


Figure 10 pt5.1 vs. Predicted Thrust

TF41 ENGINE
WFC VS. THRUST



TF41 ENGINE
WFC VS. THRUST
THRUST CALCULATED FROM REGRESSION MODEL
THRUST - 387.470*(P56) - 9785.89

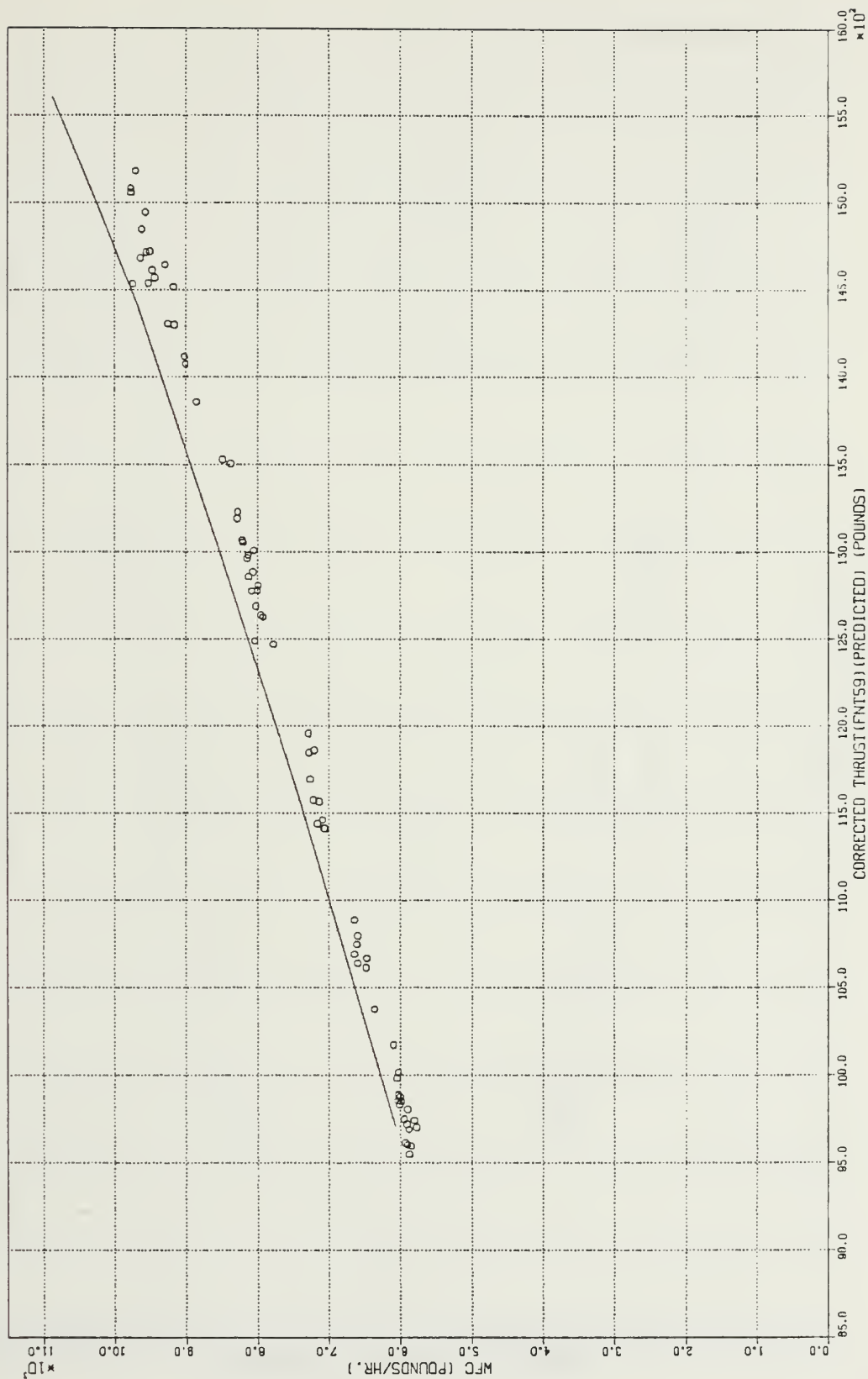


Figure 12 WFC vs. Predicted Thrust

TF41 ENGINE
WAIC VS. THRUST

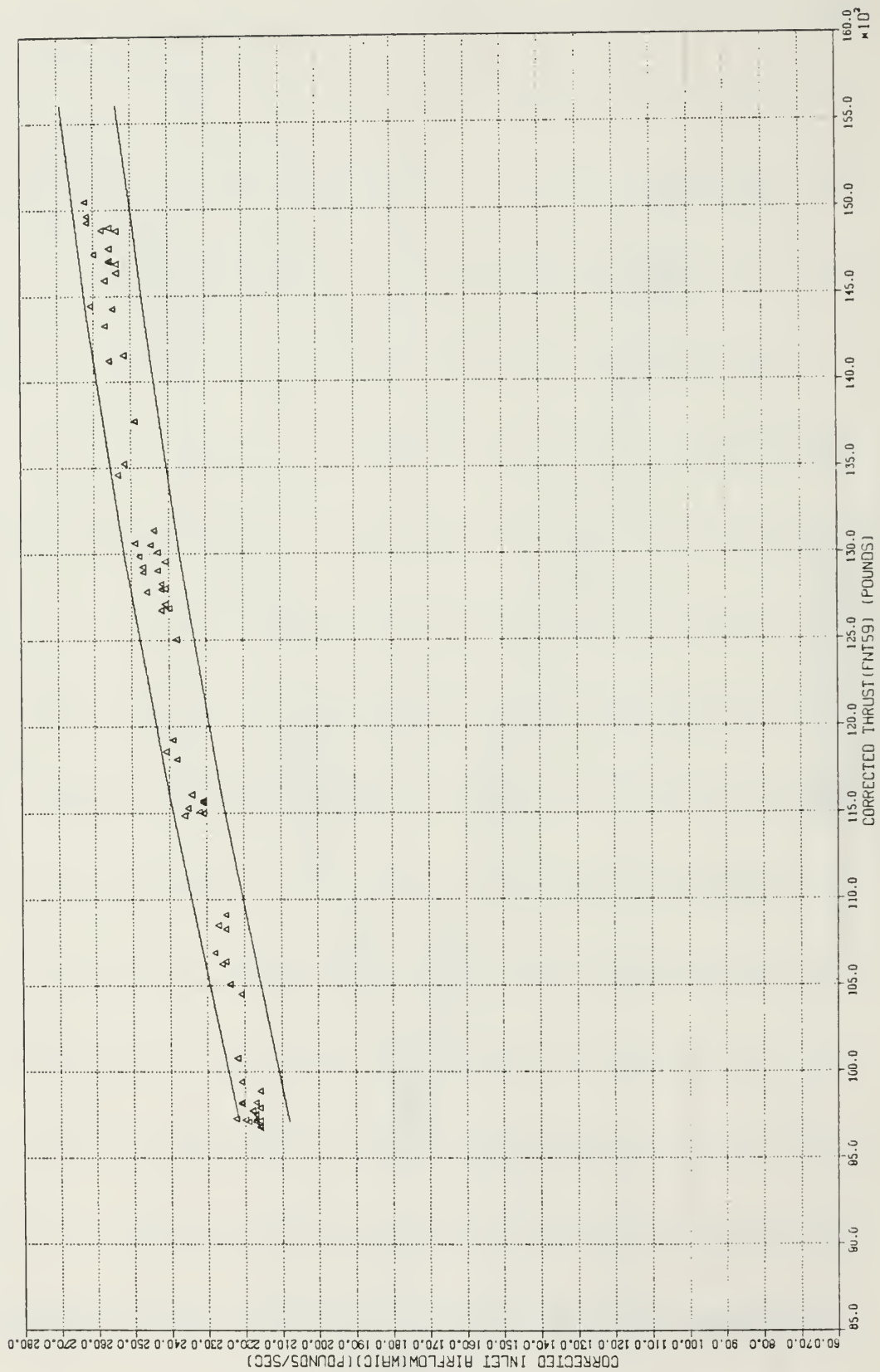


Figure 13 WAIC vs. Thrust

TF41 ENGINE
WAIC VS. THRUST
THRUST CALCULATED FROM REGRESSION MODEL
THRUST - $387.470 \times (P56) - 9785.89$

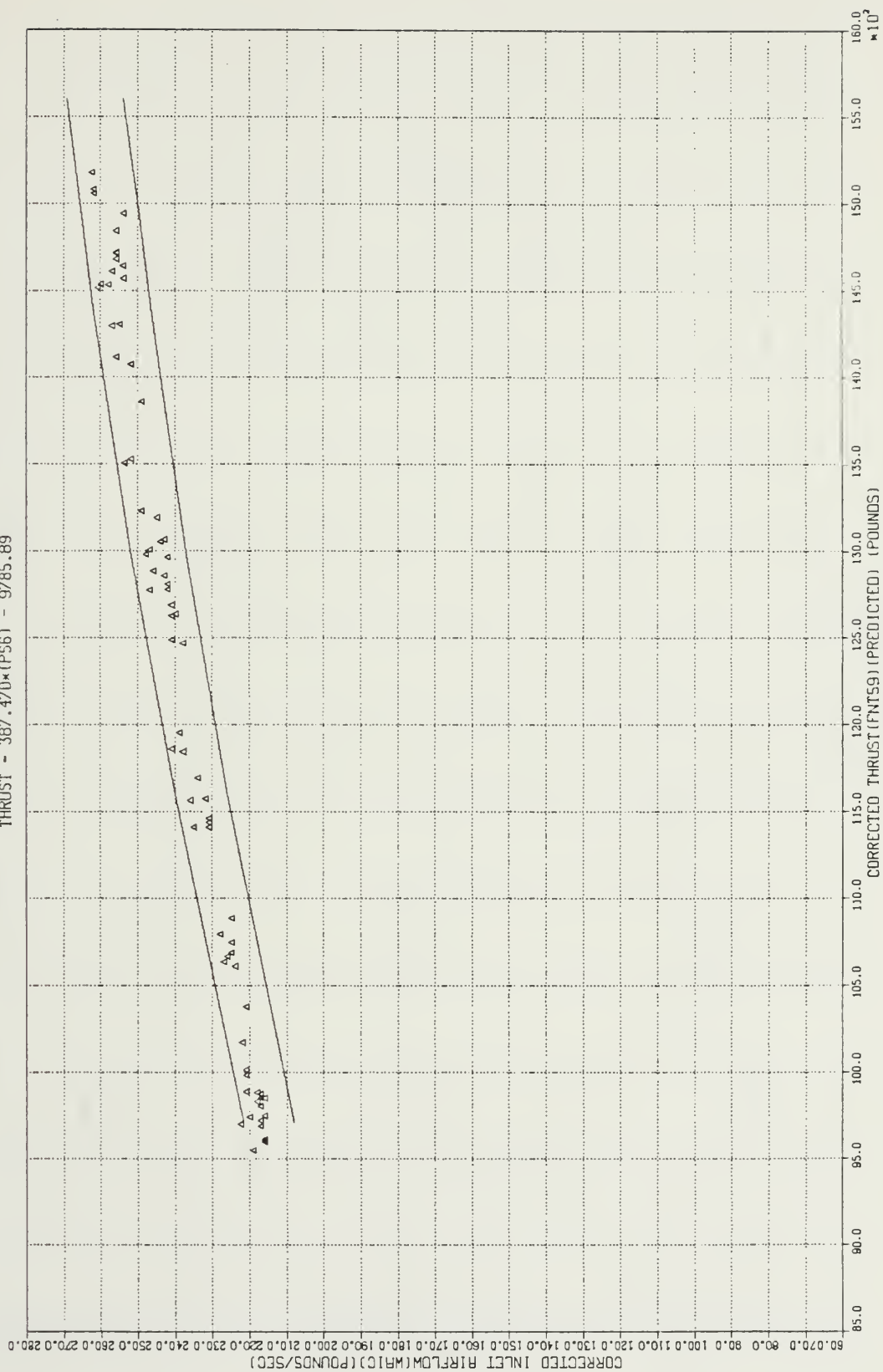


Figure 14 WAIC vs. Predicted Thrust

TF41 ENGINE
NLC VS. THRUST

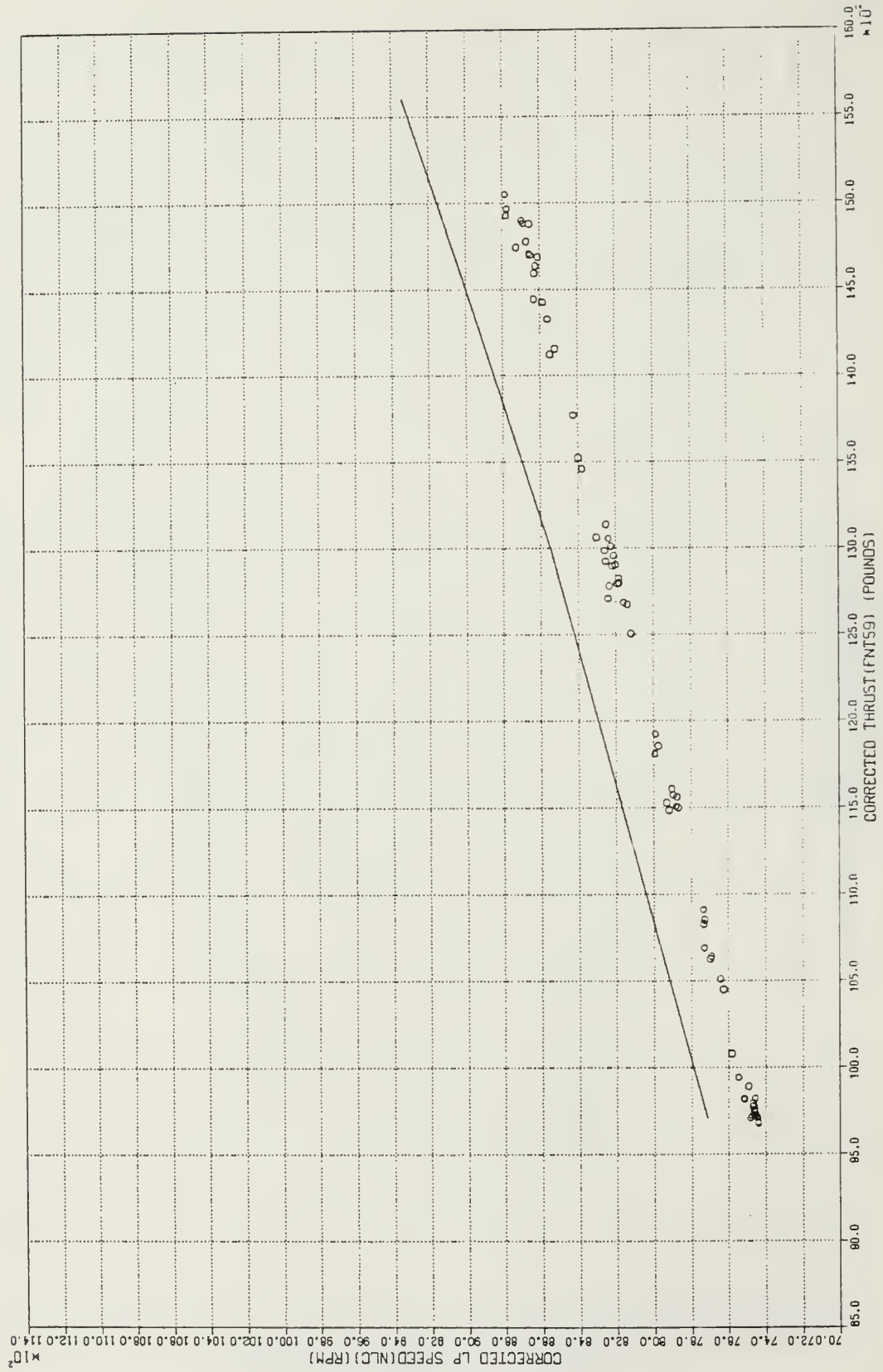


Figure 15 NLC vs. Thrust

TF41 ENGINE
 NLC VS. THRUST
 THRUST CALCULATED FROM REGRESSION MODEL
 $\text{THRUST} = 367.470 \times (\text{P56}) - 9785.89$

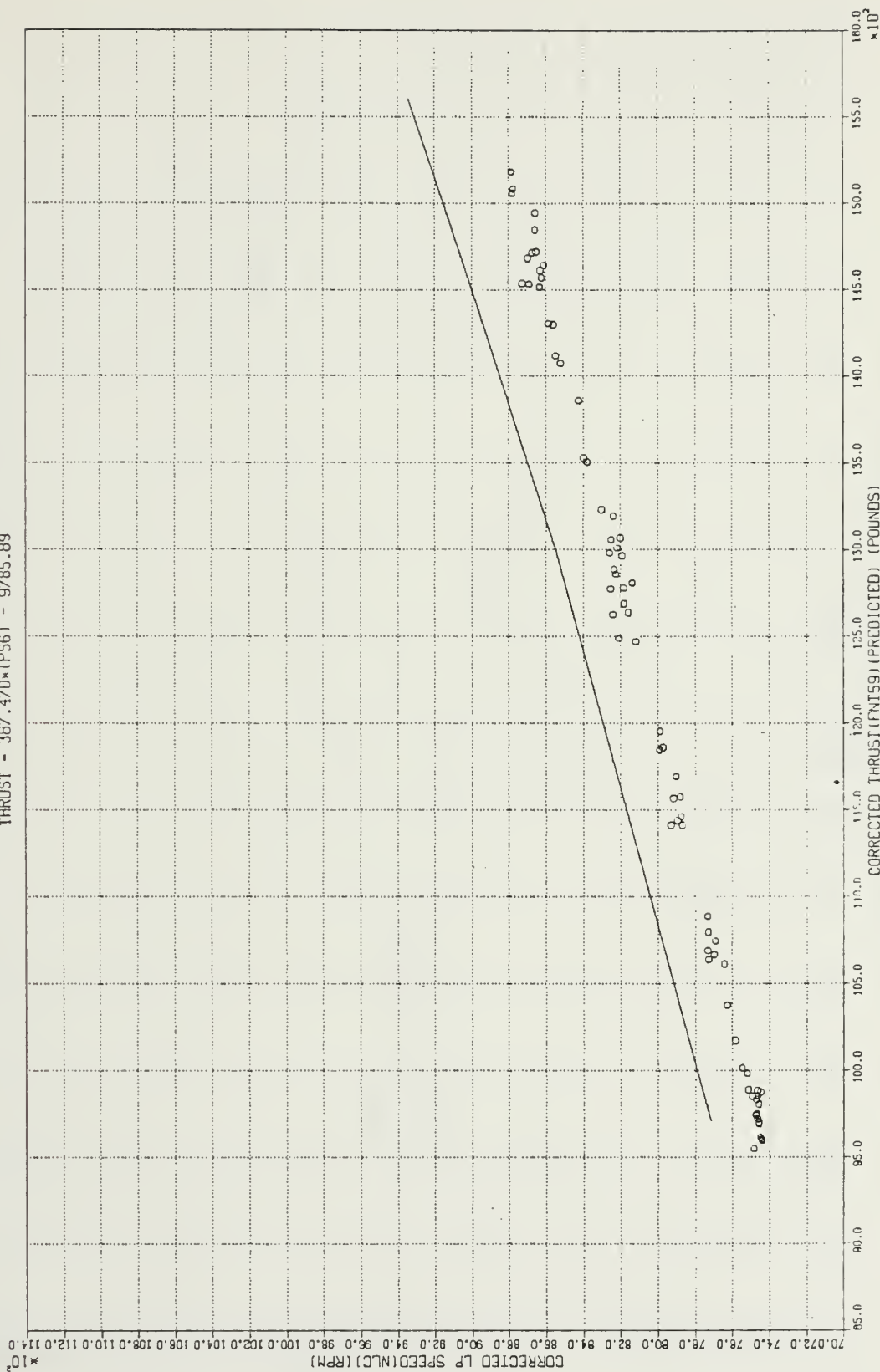


Figure 16 NLC vs. Predicted Thrust

TF41 ENGINE
NHC VS. THRUST

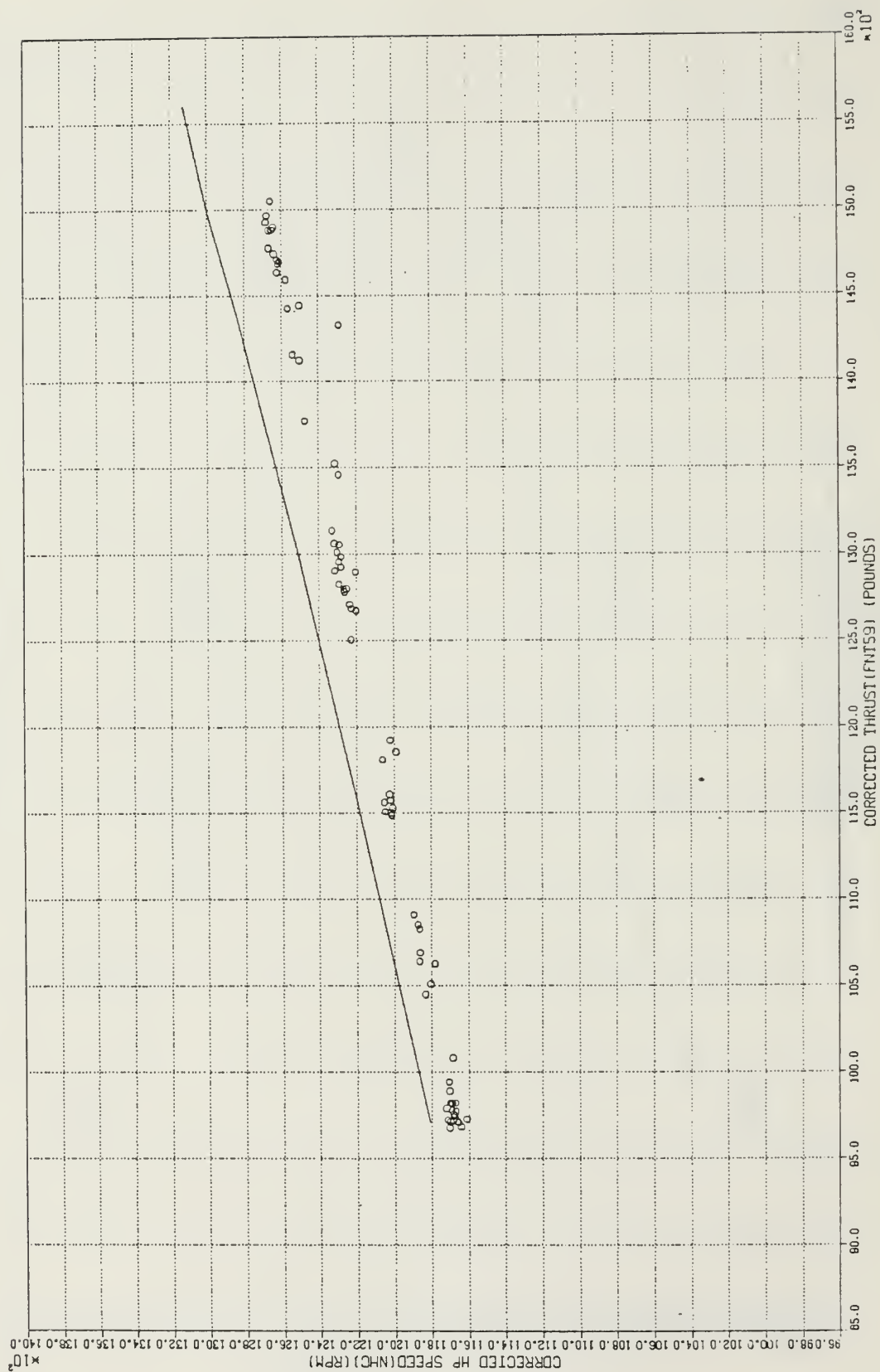


Figure 17 NHC vs. Thrust

TF41 ENGINE
NHC VS. THRUST
THRUST CALCULATED FROM REGRESSION MODEL
THRUST - 387.470*(P56) - 9785.89

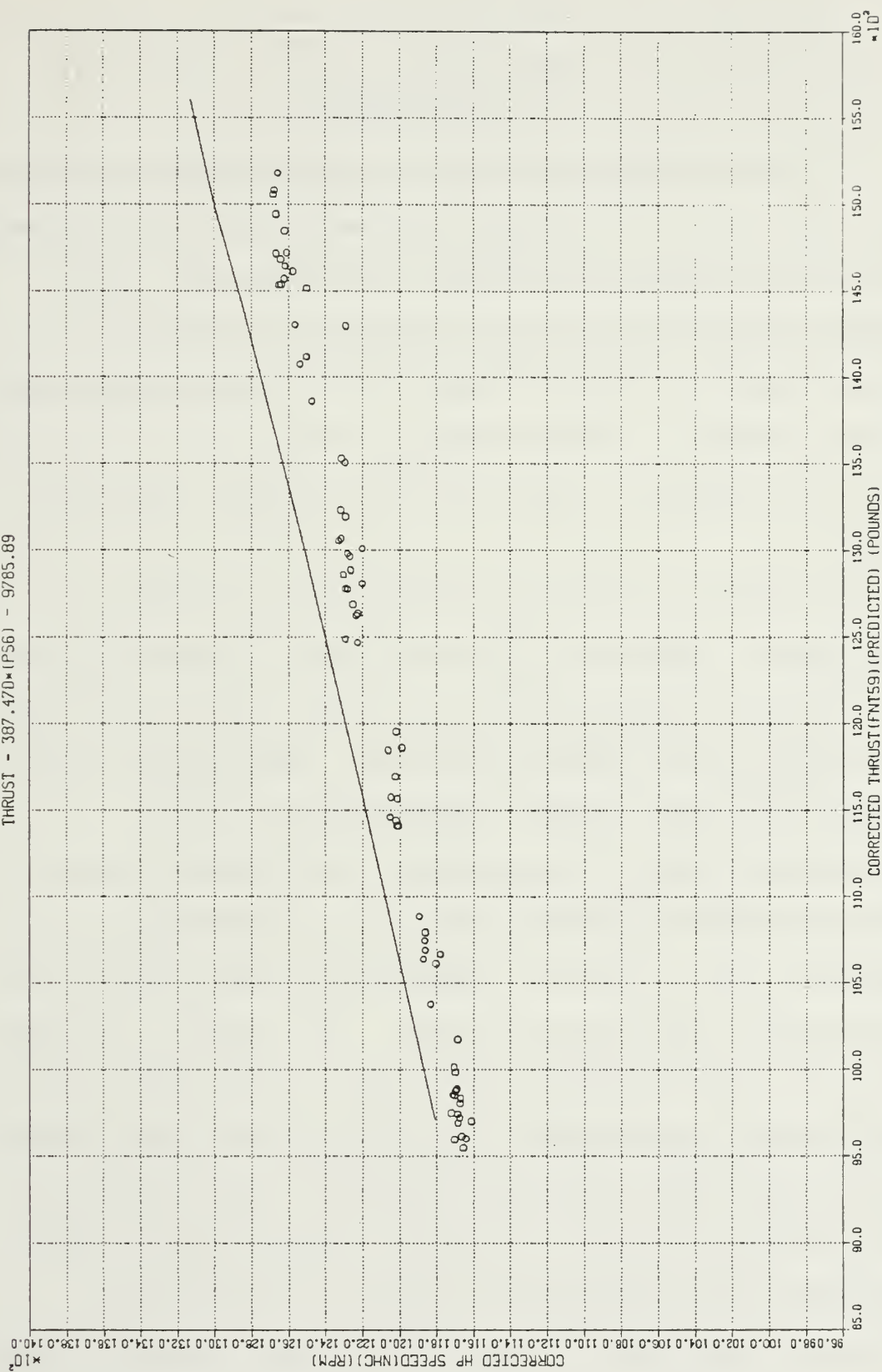


Figure 18 NHC vs. Predicted Thrust

IV. CONCLUSIONS

It is concluded that uninstalled thrust can be measured indirectly for the TF41 engine by connecting an instrumented tailpipe and measuring tailpipe static pressure. The accuracy associated with this procedure is obviously a strong function of the accuracy of the pressure measurement. For the purpose of the evaluation procedure from which these data were produced, it was found to be within acceptable limits.

The implications of this result are considered significant. Adoption of this technique, or some derivative of it, could save money and provide additional scheduling flexibility at the rework facilities. Additionally, and perhaps more significantly, implementation at the intermediate levels of maintenance could enhance flight safety by providing those levels with an additional diagnostic tool. Further, expanding the applicability of the technique could prove fruitful. For example, implementation of a trim-to-thrust technique for applicable engines could provide considerable cost savings and further enhance flight safety and operational readiness. While this was not part of our effort, in studies conducted by the USAF and NASA it was estimated that using a trim-to-thrust technique could result in savings of twelve million dollars

annually (in 1979 dollars) for the USAF F-4/J79 fleet and ten million dollars annually (in 1981 dollars) for the USAF T-38/J85 fleet, based on reductions in operating temperatures and the resulting reductions in fuel consumption, hot section parts and labor [Ref. 4].

V. RECOMMENDATIONS

Based on the results and conclusions of this study, the following recommendations are offered.

1. This technique, or a derivative, should be implemented by the depot and intermediate levels of maintenance for the TF41 engine in order to reduce cost, improve scheduling flexibility, and enhance flight safety.
2. An accuracy versus cost trade-off study should be conducted to determine an optimum instrumentation package for measuring tailpipe static pressure, keeping in mind that portability and low cost must be balanced against acceptable accuracy.
3. Similar studies should be conducted for other engines where thrust is used as the independent variable.
4. As suggested by Reference 4, further study of a trim-to-thrust technique and its implications should be conducted.

APPENDIX A

VARIATION OF THRUST WITH STATIC PRESSURE

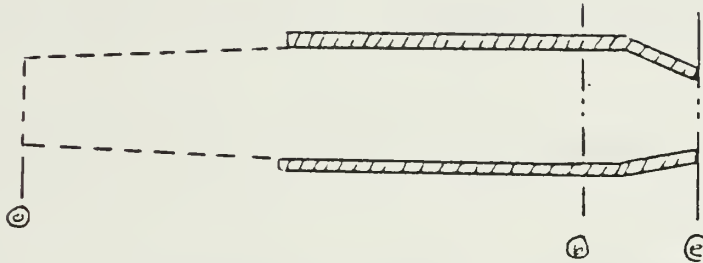


Figure A1 Schematic of Typical Engine

Thrust can be shown to vary directly with tailpipe static pressure based on a momentum analysis of the gas flow, but only after a series of simplifying assumptions are made.

Consider the following:

For one-dimensional flow, the net propulsive thrust is defined as follows [Refs. 5 and 6]:

$$F \equiv \dot{m}_e V_e - \dot{m}_o V_o + A_e (p_e - p_o) = \rho_e V_e^2 A_e - \rho_o V_o^2 A_o + A_e (p_e - p_o)$$

See Figure A1. For an ideal gas, this relationship can be modified since

$$p = \rho RT; V = Ma; a = \sqrt{\gamma RT}$$

$$\rho V^2 = \frac{P}{RT} M^2 a^2 = \frac{P}{RT} M^2 \gamma RT = p M^2 \gamma$$

$$F = p_e \gamma_e M_e^2 A_e - p_o r_o M_o^2 A_o - A_e (p_e - p_o)$$

For the static case, $M_o = 0$:

$$F = p_e \gamma_e M_e^2 A_e + A_e (p_e - p_o)$$

For choked flow, $M_e = 1$:

$$F = p_e \gamma_e A_e + p_e A_e - p_o A_e = A_e [p_e (\gamma_e + 1) - p_o]$$

Correcting for deviations from standard day conditions yields

$$F/\delta = A_e [p_e/\delta (\gamma_e + 1) - p_{SD}]$$

Thus, for a given atmospheric pressure, thrust varies only with exit pressure. Ignoring losses in the nozzle, it can be shown [Ref. 5] that

$$\frac{p_n}{p_t} = f(M_n, \gamma_n)$$

$$\frac{p_e}{p_6} = \left(\frac{p_e}{p_{te}}\right) \left(\frac{p_{te}}{p_{t6}}\right) \left(\frac{p_{t6}}{p_6}\right)$$

$$\frac{p_e}{p_6} = f(M_6, \gamma_6, \gamma_e)$$

For choked flow, M_6 and gamma will not vary significantly and p_e will be directly proportional to p_6 , ie, $p_e = k * p_6$. Corrected thrust will then vary only with p_6 as follows:

$$F/\delta = A_e [k p_6 / \delta (\gamma_e + 1) - p_{SD}] = f(p_6 / \delta)$$

The difference between this ideal analysis and the actual flow condition is, in effect, the crux of the problem. If the difference is consistent, it can be accounted for by an empirical constant. If not, another parameter (or parameters) must be introduced in order to explain this difference.

For the purposes of this project, the only assumption made in the above that might introduce a significant deviation from ideal behavior is the assumption of one-dimensionality. In a turbofan engine, conditions in the tailpipe are apt to be "very non-uniform" [Ref. 7] because of flow mixing, depending on tailpipe length, mixer design, by pass ratio, etc. For this reason, accurate measurement of total (stagnation) pressure is very difficult and use of a more common parameter, such as Engine Pressure Ratio (EPR), for the independent parameter in the project was rejected. The errors associated with tailpipe static pressure were felt to be 1) smaller and 2) more consistent (and, therefore, more predictable).

APPENDIX B

CURVE FITTING BY LEAST SQUARES

The method of least squares was employed to evaluate the relationship between tailpipe static pressure and thrust. Since inspection of the initial scatter diagram of the data indicated that thrust varied linearly, or nearly so, with ps6, and since the theoretical argument presented in Appendix A indicates that such a relationship is physically plausible, a model of the form $Y = \bar{a}_1 + a_2 * X + e$ was chosen.

Calculation of the estimated coefficients a_1 and a_2 was accomplished by a standard technique wherein

$$a_2 = \frac{\sum \frac{1}{n} (x_i - \bar{x}) (y_i - \bar{y})}{\sum \frac{1}{n} (x_i - \bar{x})^2} = \frac{\sum \frac{1}{n} x_i y_i - \frac{1}{n} \sum \frac{1}{n} x_i \sum \frac{1}{n} y_i}{\sum \frac{1}{n} x_i^2 - \frac{1}{n} (\sum \frac{1}{n} x_i)^2}$$

$$a_1 = \bar{y} - a_2 \bar{x}$$

See Reference 8 and the FORTRAN program in Appendix C for details.

Some assumptions are implicitly made when employing such a model. First, the values of the independent parameter are assumed to be measured without error; that is, all errors are assumed to be in the dependent variable. Secondly, the error term, e , is assumed to be a random variable with zero

mean and constant variance. And thirdly, the errors are assumed to be normally distributed.

Several techniques were employed in testing the validity of these assumptions. The first of these, and perhaps the most compelling intuitively, was an examination of a plot of the residuals (the difference between the observed and fitted values) versus the independent variable (see Figure B1). It can be readily seen that, with the exception of two outliers, all the data are displayed in a horizontal band with an average value of zero and a reasonably constant variance.

The assumption of normality in the distribution of errors was proven to be reasonable by use of the Chi-square statistical test [Ref. 9]. The residuals were divided into six regions as shown in Table B1. Two degrees of freedom were utilized in estimating the frequency classes and the standard deviation from the mean, leaving four degrees of freedom for the test. Employing the Normal Distribution table in Appendix 3 of Reference 9, the expected values of the frequency of errors in each region were determined. The results are shown in Table B1.

At the 95% Confidence Level, Chi-square equals 9.49 for four degrees of freedom and since the calculated value is less than the significant value, the hypothesis that the errors are, in fact, normally distributed was accepted.

TF41 ENGINE
 LINEAR REGRESSION MODEL
 RESIDUALS VS. PS6

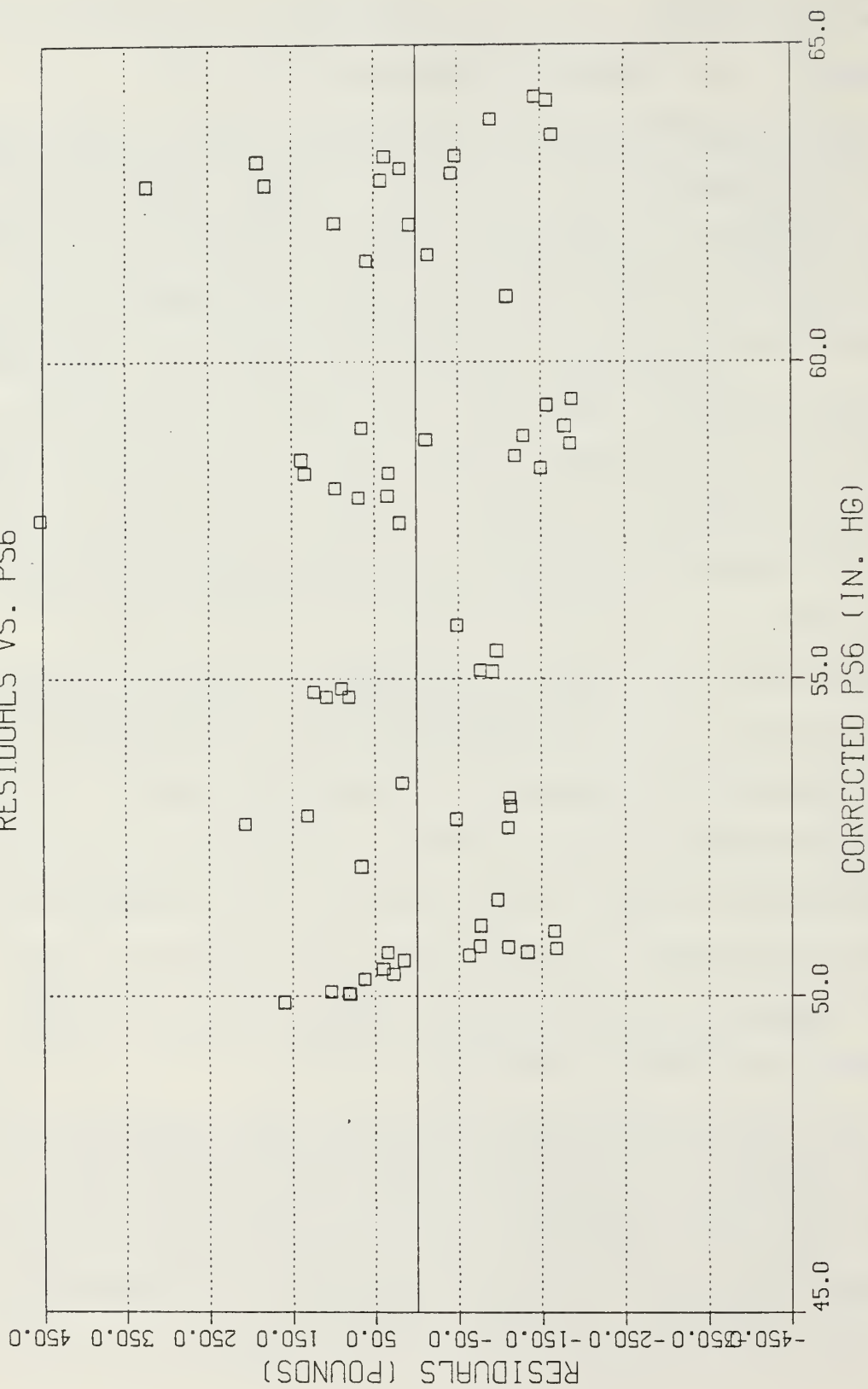


Figure B1 Residuals vs. ps6

TABLE B1

REGION	EXPECTED VALUE	OBSERVED VALUE	CHI-SQUARE
1 (0-67)	15.2775	15	0.005
2 (67-134)	11.55	13	0.182
3 (134+)	10.6725	8	0.669
4 (0-(-67))	15.2775	12	0.703
5 ((-67)-(-134))	11.55	16	1.715
6 ((-134)-)	10.6725	11	0.010
			3.284

As a further quantitative measure of the degree of association between the two variables, the sample correlation coefficient was calculated. This parameter is defined as follows:

$$r = \frac{\sum_n (x_i - \bar{x})(y_i - \bar{y})}{\left\{ \sum_n (x_i - \bar{x})^2 \right\}^{1/2} \left\{ \sum_n (y_i - \bar{y})^2 \right\}^{1/2}}$$

The value of r was found to be close to unity ($r=0.9976$).

The validity of the sample correlation coefficient was tested by hypothesizing that the population correlation coefficient was equal to zero. That is,

$$H_0: \rho = 0$$

$$H_a: \rho \neq 0$$

The standardized random variable t was used for the test, where

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

This variable is t -distributed with $n-2$ degrees of freedom. At the 95% Confidence Level and with 73 degrees of freedom, the hypothesis that ρ equalled zero was rejected since $t=126.2$ which is greater than the significant value of $t=1.993$ (see Appendix 5 of Ref. 9).

A confidence interval was established about the value of ρ (the population correlation coefficient) based on the calculated value of r (the sample correlation coefficient). At the 95% Confidence Level

$$0.9951 \leq \rho \leq 0.9993$$

See Chapter 8-5 of Ref. 9 for details.

And finally, confidence intervals were placed about the model's mean line, both at the 95% and 99% levels, to help clarify the model's ability to correctly predict thrust. See Figures B2 and B3.

TF41 ENGINE
 LINEAR REGRESSION MODEL
 CORRECTED THRUST VS. CORRECTED PS6
 $Y = 389.246 \times X - 9874.07$

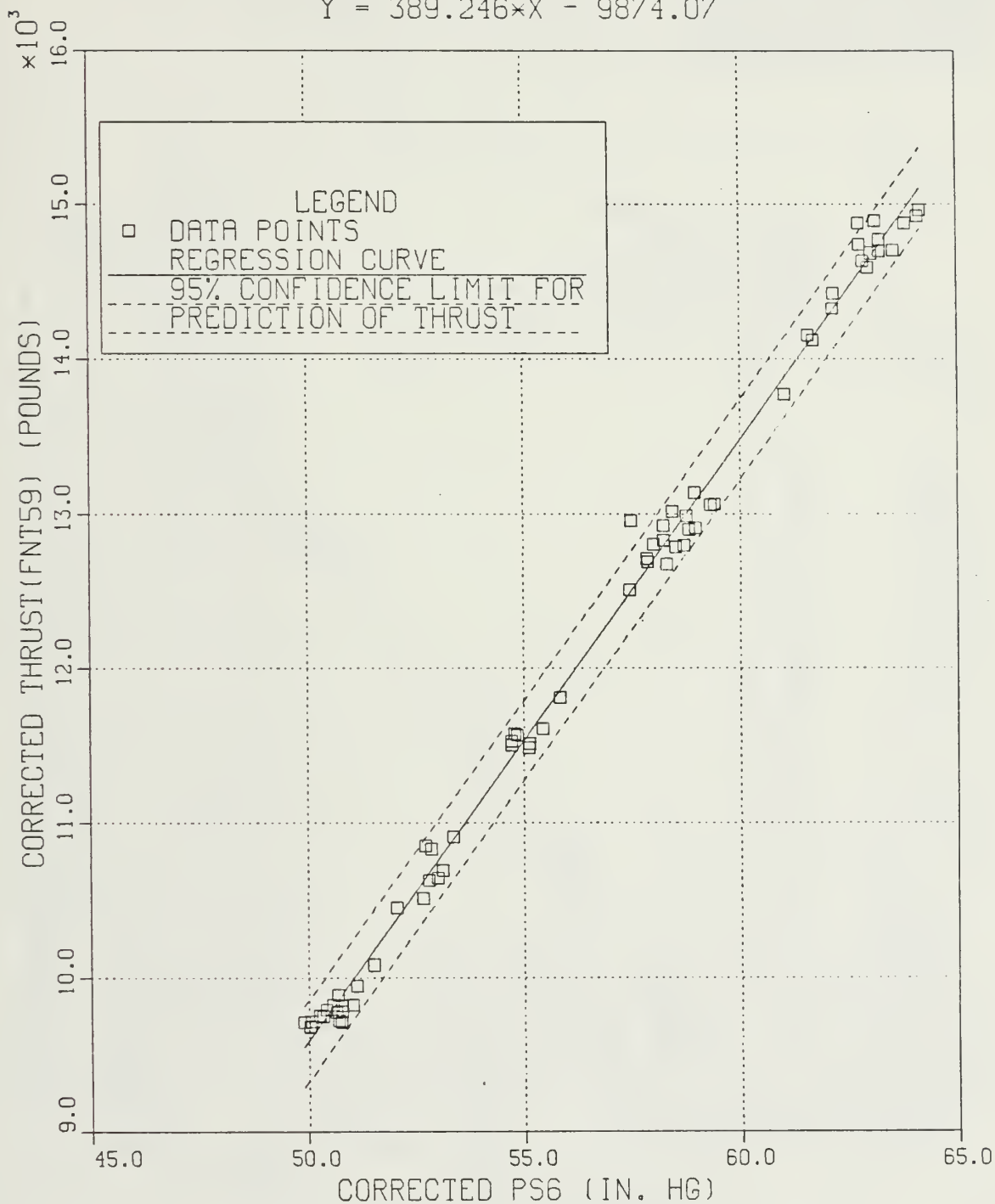


Figure B2 Thrust vs. ps6

TF41 ENGINE
 LINEAR REGRESSION MODEL
 CORRECTED THRUST VS. CORRECTED PS6
 $Y = 389.246 \times X - 9874.07$

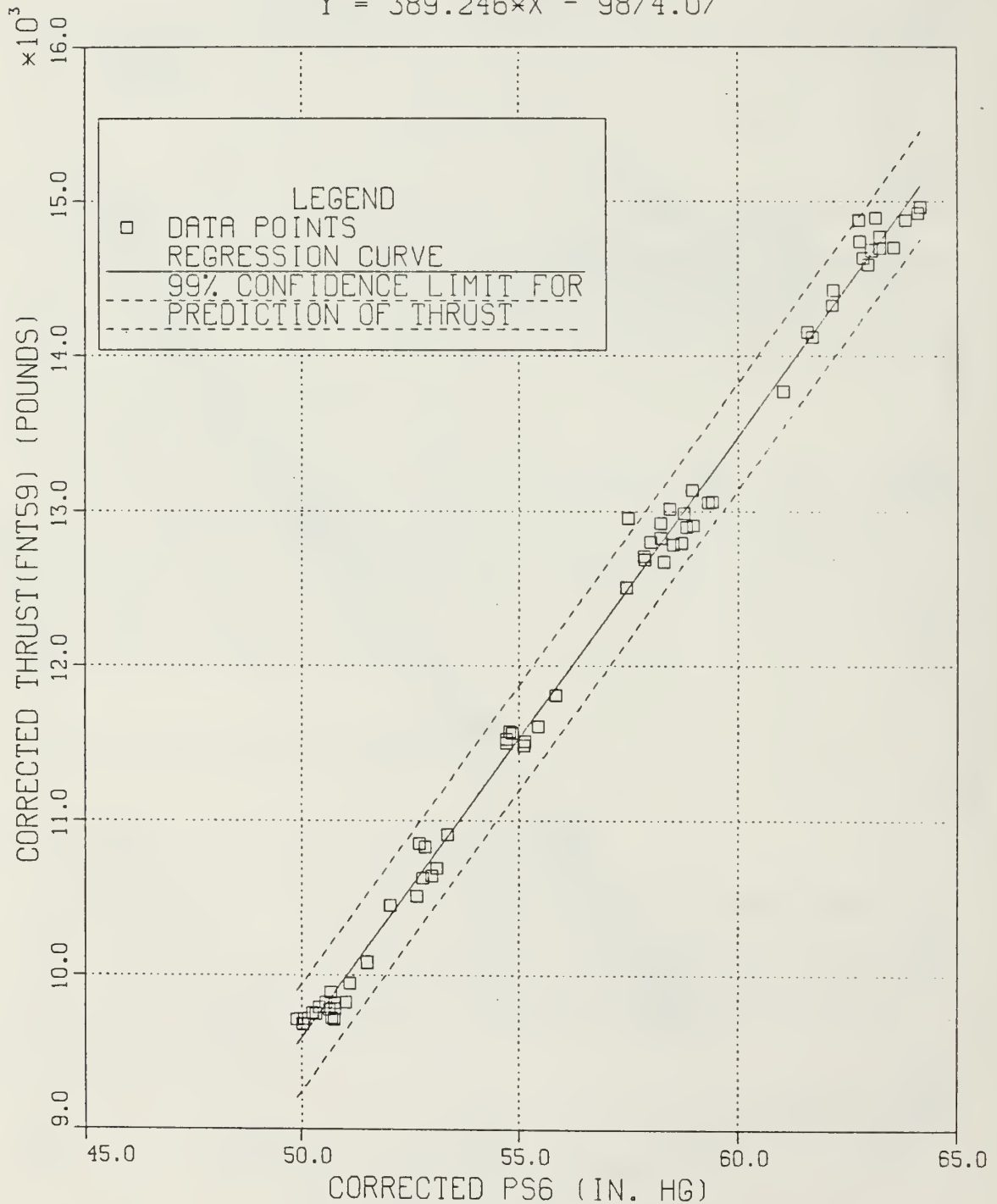


Figure B3 Thrust vs. ps6 (99% Conf.)

TF4000290
TF400300
TF400310
TF400340
TF400360
TF400370
TF400380
TF400390
TF400400
TF400420
TF400430
TF400440
TF400450
TF400470
TF400480

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```

KVPS6=.9991434+6.805E-5*W-1.7261E-6*W**2+2.1877E-8*W**3-1.2357E-10
1*W**4+2.5516E-13*W**5
CPPS6=1.003923-6.76411E-5*DAT(5,J)
X1(L,J) = (DAT(9,J)*KVPS6*CPPS6/DELTA)
      (FG/FN) F/DELTA
CPFN=1.00708-1.2332E-4*DAT(5,J)+1.41685E-8*DAT(5,J)**2
KVFN=1.000316+9.738E-6*W+1.3439E-8*W**2
DAT(11,J)=DAT(10,J)*KVFN*CPFN/DELTA
DAT(12,J)=DAT(11,J)-248.25+.0414892*DAT(11,J)
Y1(L,J)=DAT(12,J)
Y1(L,J) = (DAT(12,J)*1.00251)
GO TO 10
CONTINUE
DO 310 I = 1,9
DO 320 J = 1,8
  IF(J.EQ.1) X(I) = X1(I,J)
  IF(J.EQ.2) X(I+9) = X1(I,J)
  IF(J.EQ.3) X(I+18) = X1(I,J)
  IF(J.EQ.4) X(I+27) = X1(I,J)
  IF(J.EQ.5) X(I+36) = X1(I,J)
  IF(J.EQ.6) X(I+45) = X1(I,J)
  IF(J.EQ.7) X(I+54) = X1(I,J)
  IF(J.EQ.8) X(I+63) = X1(I,J)
  IF(J.EQ.1) Y(I) = Y1(I,J)
  IF(J.EQ.2) Y(I+9) = Y1(I,J)
  IF(J.EQ.3) Y(I+18) = Y1(I,J)
  IF(J.EQ.4) Y(I+27) = Y1(I,J)
  IF(J.EQ.5) Y(I+36) = Y1(I,J)
  IF(J.EQ.6) Y(I+45) = Y1(I,J)
  IF(J.EQ.7) Y(I+54) = Y1(I,J)
  IF(J.EQ.8) Y(I+63) = Y1(I,J)
CONTINUE

```

```

TF4000490
TF4000500
TF4000510
TF4000520
TF4000660
TF4000670
TF4000680
TF4000690
TF4000700
TF4000710
TF4000710
TF4000720
TF4000810

```

C
C
C

C

C

C

200

20

300

320
310

1120

1130

1140


```

810 CONTINUE = 0.0
   SAYBAR = 0.0
   DO 800 I=1,Z
     AYBAR(I) = Y3(I) - YBAR(I)
     SAYBAR(I) = SAYBAR(I) + AYBAR(I)
     IF ((AYBAR(I).GT.0.0).AND.(AYBAR(I).LT.67.))O(1)=O(1)+1.
     IF ((AYBAR(I).GE.67.))AND.(AYBAR(I).LT.134.))O(2)=O(2)+1.
     IF ((AYBAR(I).GE.134.))O(3)=O(3)+1.
     IF ((AYBAR(I).LE.0.0).AND.(AYBAR(I).GE.-67.))O(4)=O(4)+1.
     IF ((AYBAR(I).LT.-67.))AND.(AYBAR(I).GE.-134.))O(5)=O(5)+1.
     IF ((AYBAR(I).LT.-134.))O(6)=O(6)+1.
800 CONTINUE
   CHI = 0.0
   CHISQ = 0.0
   DO 910 I=1,6
     IF ((I.EQ.1).OR.(I.EQ.4))E=13.
     IF ((I.EQ.2).OR.(I.EQ.5))E=10.
     IF ((I.EQ.3).OR.(I.EQ.6))E=10.
     CHI = (O(I)-E)*2/E
     CHISQ = CHI+CHISQ
910 CONTINUE
   WRITE(6,665)CHISQ
665 FORMAT(1X,'CHISQUARED = ',F10.2)
   WRITE(6,666)(O(I),I=1,6)
666 FORMAT(1X,6F10.2)
   WRITE(6,667)SAYBAR
667 FORMAT(1X,F10.2)
C*****
C***** CALCULATE THE CONFIDENCE INTERVAL FOR THE MEAN CURVE*****
C*****
   DO 700 I=1,Z
     SCSUM = SCSUM+X3(I)**2
     SUMSQ = SUMSQ+X3(I)**2
700 CONTINUE
   MX3 = SCSUM/FLOAT(Z)
   SCSUM = SCSUM**2
   DO 710 I=1,Z
     SSYBAR(I) = SS*(1./FLOAT(Z)+((X3(I)-MX3)**2/(SUMSQ-(SCSUM/
$FLOAT(Z))))))
     SYBAR(I)=SQRT(SSYBAR(I))
710 CONTINUE
   DO 400 I=1,Z
     MYBAR(I) = YBAR(I) + 1.993*SYBAR(I)
400 CONTINUE
   DO 410 I=1,Z
     MYBAR(I) = YBAR(I)-(1.993*SYBAR(I))
410 CONTINUE
   WRITE(6,600)((X3(I),SYBAR(I)),I=1,Z)
   WRITE (6,605)MX3

```

```

605  FORMAT(IX,'PS6 MEAN = ',F10.2)
C*** CALCULATE LINEAR CORRELATION COEFFICIENT *****
S1 = 0.0
S2 = 0.0
S3 = 0.0
S4 = 0.0
S5 = 0.0
S6 = 0.0
S7 = 0.0
DO 420 I = 1,Z
  S1 = S1 + X3(I)*Y3(I)
  S2 = S2 + X3(I)
  S3 = S3 + Y3(I)
  S4 = S4 + X3(I)**2
  S5 = S5 + Y3(I)**2
420 CONTINUE
S6 = S2**2
S7 = S3**2
R = (S1-(S2*S3)/FLOAT(Z))/((S4-S6/FLOAT(Z)))
C*** CALCULATE THE COEFFICIENT OF DETERMINATION *****
RR = (S1-(S2*S3)/Z)**2/((S4-S6/Z)*(S5-S7/Z))
C*** CALCULATE THE CONFIDENCE INTERVAL FOR THE SLOPE OF THE MODEL *****
SA2 = Sqrt((S5-S6/FLOAT(Z)))
LCSLOP = A(2)-1.993*SA2
UCSLOP = A(2)+1.993*SA2
C = 1
IF(C.EQ.2) GO TO 50
C*** CALCULATE THE 95% PREDICTION INTERVAL FOR YNEXT *****
DO 421 I = 1,Z
  SSYN(I) = SS*(1.+1./FLOAT(Z))+((X3(I)-MX3)**2)/(S4-S6/FLOAT(Z))
  SYN(I) = Sqrt(SSYN(I))
  ASYN(I) = 1.993*SYN(I)
  UPLYN(I) = YBAR(I)+1.993*SYN(I)
  LPLYN(I) = YBAR(I)-1.993*SYN(I)
421 CONTINUE
  WRITE(6,600)((X3(I),ASYN(I)),I=1,Z)
C
C
C*** OUTPUT *****
WRITE(6,610)IER
WRITE(6,620)A
WRITE(6,630)S
WRITE(6,640)SS
WRITE(6,645)SIGMA
WRITE(6,655)R
WRITE(6,656)RR

```



```

C***CALCULATE THE 99% PREDICTION INTERVAL FOR YNEXT*****
50 DO 422 I=1,Z
   SSYN(I) = SS*(1.+1./FLOAT(Z)+((X3(I)-MX3)**2)/(S4-S6/FLOAT(Z)))
   SYN(I) = Sqrt(SSYN(I))
   ABSYN(I) = 2.643*SYN(I)
   UPLYN(I) = YBAR(I)+2.643*SYN(I)
   LPLYN(I) = YBAR(I)-2.643*SYN(I)
422 CONTINUE
   WRITE(6,600)((X3(I),ABSYN(I)),I=1,Z)
60 TO 51
52 CALL LINES('99% CONFIDENCE LIMIT FOR$', PKRAY,3)
60 TO 53
C56 CALL MESSAGE('FIGURE 1. THURST VS. PS6$',100,0.0,-1.0)
56 CALL MESSAGE('FIGURE B3. THURST VS. PS6$',100,0.0,-1.0)
60 TO 57
C***** SET PLOT AREA *****
54 CALL CCMPS
CALL BLOWUP(.9)
C 54 CALL TEK618
C CALL HWRGT('AUTO')
C CALL PAGE(11.0,8.5)
C CALL PAGE(8.5,11)
C CALL NCBDR
C CALL PHYSOR(1.0,1.0)
CALL AREA2D(8.5,5.0)
CALL XNAME('CORRECTED PS6 (IN. HG)$',100)
CALL YNAME('RESIDUALS (POUNDS)$',100)
CALL HEADIN('TF41 ENGINE$',100,1.0,3)
CALL HEADIN('LINEAR REGRESSION MODEL$',100,1.0,3)
CALL HEADIN('RESIDUALS VS. PS6$',100,1.0,3)
CALL GRAF(45.,5.,65.,-450,100,450)
C CALL RLVEC(45,0,65,0,0000)
CALL MESSAGE('FIGURE 2. RESIDUALS VS. PS6$',100,0.0,-1.0)
CALL MESSAGE('FIGURE B1. RESIDUALS VS. PS6$',100,0.0,-1.0)
CALL DGT
CALL GRID(1,1)
CALL RESET('ALL')
CALL CURVE(X3,AYBAR,Z,-1)
CALL ENDPL(0)
CALL DCNEPL
STOP
C 500 FORMAT(2A4,8F8.2)
610 FORMAT('1. COEFFICIENTS FOR THE POLYNOMIAL CURVE FIT ARE: ',/1X,
620 'FURMAT(1X, COEFFICIENTS FOR THE POLYNOMIAL CURVE FIT ARE: ',/1X,
630 'FURMAT(1X, THE SUM OF THE SQUARES OF THE DIFFERENCES IS: ',/1X,

```

TF400350

```

640 FORMAT(1X, 'THE RESIDUAL ERROR VARIANCE IS: ', 1X, 'SS = ', E12.6//)
645 FORMAT(1X, 'THE STANDARD DEVIATION IS: ', 1X, 'SIGMA = ', E12.6//)
655 FORMAT(1X, 'THE LINEAR CORRELATION COEFFICIENT IS: ', 1X,
      &      'R = ', E12.6//)
656 FORMAT(1X, 'THE COEFFICIENT OF DETERMINATION IS: ', 1X,
      &      'RR = ', E12.6//)
657 FORMAT(1X, 'THE CONFIDENCE INTERVAL FOR THE SLOPE OF THE MODEL '
      &      'IS: ', 1X, E12.6, ' TO ', E12.6//)
      &      END
      &      REAL FUNCTION F(K,X)
      &      INTEGER K
      &      REAL X,RK
      &      RK = X
      &      F = X**(RK-1.0)
      &      RETURN
      &      END

```

TF401380

APPENDIX D

DATA

The following is a portion of the data received from the Naval Air Rework Facility, Jacksonville, Florida. The corrected thrust (FNT(59)) was measured in a calibrated test cell and utilized as is. It should be noted that engine S/N 142618, a "correlation engine", had been used at the start of our project to verify the proper calibration of the test cell. Tailpipe static pressure was measured by utilizing a "slave" tailpipe instrumented with a series of pressure transducers. See Figure D1 and D2.

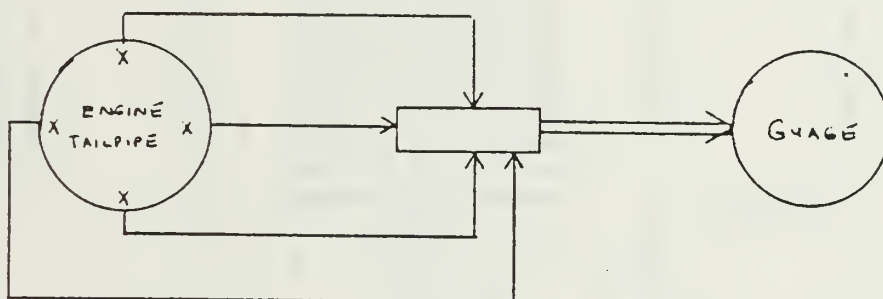


Figure D1 Instrumentation Setup

The result (ps6) is an average value. Corrected tailpipe static pressure (ps6c) resulted from applying the same correction factor utilized in correcting other pressure values in reference 3. See Appendix C.

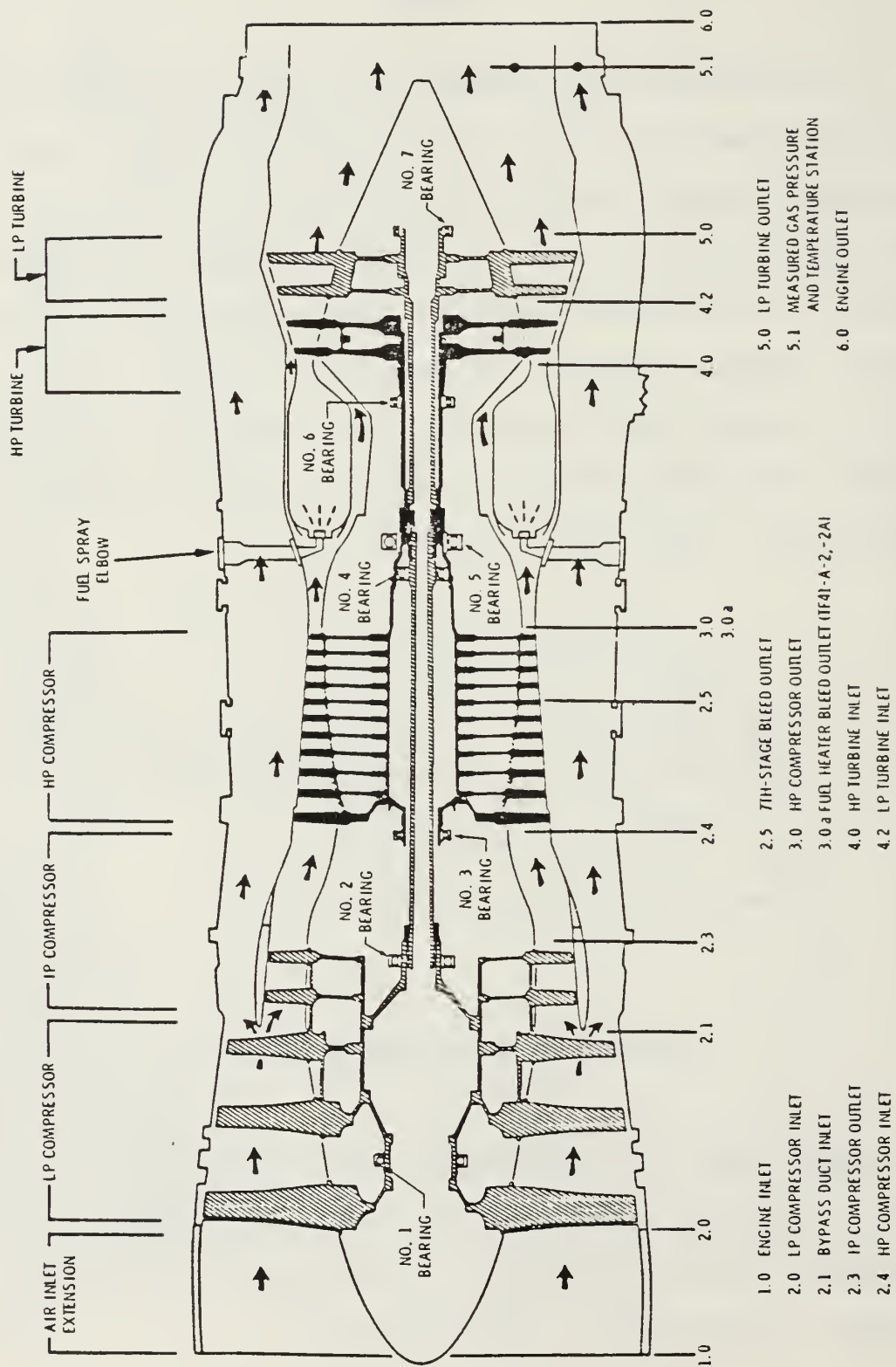


Figure D2 TF41 Engine Station Schematic

ENGINE S/N	FNT (59)	PS6	PS6C
	(lbf)	(in-Hg)	(in-Hg)
141481	9717	50.4	50.05
	11575	55.1	54.79
	12802	58.3	58.01
	14895	63.4	63.14
	9685	50.2	50.05
	10828	53.0	52.85
	12687	58.0	57.87
	14696	63.3	63.24
141525	9678	50.4	50.02
	11563	55.2	54.84
	12828	58.6	58.24
	14775	63.6	63.23
	9792	50.8	50.42
	10450	52.4	52.04
	12506	57.8	57.44
	14633	63.2	62.85
142634	9710	50.2	49.90
	11530	55.0	54.71
	12926	58.5	58.23
	14742	63.0	62.77
141954	9755	50.4	50.27
	11503	54.8	54.71
	13013	58.5	58.44
	14156	61.6	61.58
	9750	50.3	50.34
	10852	52.6	52.72
	12711	57.7	57.84
	14425	62.0	62.17

141427	9724	50.6	50.69
	11510	55.0	55.13
	12906	58.8	58.98
	13772	60.8	61.03
	9712	50.81	50.75
	10641	53.06	52.99
	12796	58.67	58.71
	14706	63.52	63.57
141972	9822	50.4	51.02
	11487	54.4	55.11
	13063	58.6	59.41
	14925	63.2	64.11
	9817	50.50	50.78
	10692	52.81	53.12
	12783	58.13	58.51
	14963	63.65	64.17
141440	9943	49.98	51.11
	11812	54.57	55.84
	12987	57.41	58.76
	14125	60.21	61.69
	10081	50.00	51.51
	10627	51.20	52.79
	12897	57.00	58.83
	14328	60.20	62.16
142633	9822	49.9	50.56
	13137	58.1	58.95
	14878	62.6	63.82
	9889	50.3	50.68
	10912	52.9	53.36
	12957	57.0	57.49
	14685	62.4	63.04

141257	9778	50.4	50.77
	11610	55.0	55.44
	13056	58.8	59.31
	14880	62.2	62.76
	9773	50.6	50.63
	10511	52.6	52.65
	12675	58.2	58.31
	14592	62.8	62.96
142618	9722.6	50.4	50.4
	11925.0	56.0	56.11
	13526.5	60.0	60.18
	15047.1	64.2	64.43
	9729.0	50.3	50.3
	11858.4	55.8	55.87
	13459.3	60.0	60.11
	14441.8	62.6	62.72

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thrust from tailpipe
static pressure.

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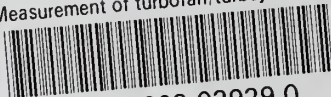
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